A Reacfin White Paper in Quantitative Finance:

Modeling negative interest rates with Free Boundaries SABR

Approaches for model assessment and validation

by Dr. Sebastien de Valeriola, Wim Konings and François Ducuroir

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INTRODUCTION

Confronted with the current low-rates environment, many fixed income derivatives market professionals (investment banks, market makers, hedge funds, etc.) are reviewing possible solutions to adapt their derivatives pricers

It is particularly the case of those models assuming stochastic volatilities under SABR processes.

A recent article published by the quant teams of Numerix[1] proposes innovative solutions to go beyond the usual trivial approaches or "rates shifting" solutions.

In this white paper we summarize our understanding of the article and propose an initial review of the main points of attention one should consider in the context of model assessment and its validation.

[1] See "The Free Boundary SABR: Natural Extension to Negative Rates" by Alexandre Antonov, Michael Konikov and Michael Spector at Numerix published on January 28, 2015 and available on http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2557046

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KEY TAKE-AWAYS OF THIS WHITE PAPER

We consider that from a theoretical point of view the content of the article by Alexandre Antonov, Michael Konikov and Michael Spector is sound and achieves its goal. It is thus indeed an innovative method worth consideration in a field of key importance where only limited alternatives are available to practitioners.

For the purpose of assessing, implementing or validating such model, we suggest the following key points of attention:

- The calibration of the Beta parameter which might prove more impactful than in traditional SABR models
- The assessment of impact of the spike in distributions resulting from the summing of two probability densities (corresponding to solutions to the Fokker-Planck equation with "opposite" signs).
- The assessment of impact of the numerical approximation methods used
- Other tests and modeling recommendations to evaluate the stability of the model results.

Practically we would suggest in the context of a model assessment or of a validation process to start by:

- Replicating the Monte Carlo Scheme used in the paper to highlight the effectiveness of the proposed approach
- Performing a comparison with distributions resulting from the more simple shifted SABR model
- Performing a stability assessment of the derivatives Greeks obtained using the proposed model

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CONTENTS

IMPORTANT NOTE:

This White Paper is not intended to be a detailed scientific review of a modeling approach in quantitative finance. It must thus be considered only for what it is: "food for thoughts" on the article written by Alexandre Antonov, Michael Konikov and Michael Spector, which would further deserve additional more detailed assessment.

1. THE PROBLEM OF LOW/NEGATIVE RATES

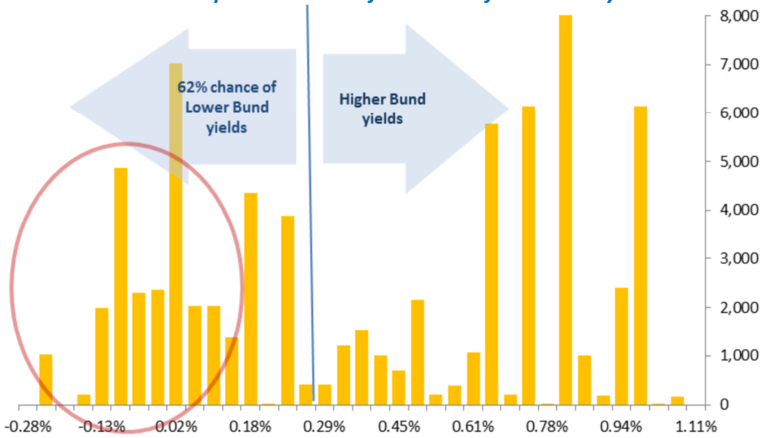
In the recent period, financial markets operated in low or even negative rates for short maturities. This situation has shown to be problematic in the context of option valuations.

Specifically one could recently observe significant investor demand for optional strategies with zero- or negative strikes.

As suggested by the research teams of Royal Bank of Scotland[2], an interesting hint hereof is to look at exchange-traded options on the German Bund on EUREX. These options provide some insight in the investors positioning if we assume that the price of each contract provide indication about market sentiment in terms of where rates could go.

Exhibit 1 below[3] plots the open interest for June 2015 contracts by strike[4] as of early February.

***Exhibit 1** Open Interest in Jun15 options on Bund futures as of 06 February 2015*



Source: Royal Bank of Scotland

One will observe the material open interest around or below zero-rates strikes.

The SABR, a stochastic volatility model widely used by practitioners to price interest rates derivatives, imposes positive rates assumption.
The dynamics of the SABR model is given by[5]

$$\begin{aligned} dF_j(t) &= V(t)F_j(t)^{\beta}\,dZ_j^j(t), \\ dV(t) &= \epsilon V(t)\,dW^j(t), \\ V(0) &= \alpha, \end{aligned}$$

where Z_j^j and W^j are Q^j-standard Brownian motions with

$$dZ_j^j(t)\,dW^j(t) = \rho\,dt,$$

[2] See "Rates Volatility Weekly Europe – Don't get caught short gamma", 6-Feb-2015, pp 5-6, Clement Mary-Dauphin, European Rates Research at Royal Bank of Scoland

[3] Id.

[4] Strikes are actually converted in yield format rather than bund prices

[5] See also Brigo-Mercurio "Interest Rate Models -Theory and Practice", ISBN-13: 978-3540221494, Springer, August 2006, p. 508

Compared to other stochastic volatility models, volatility does not "mean revert" so it is only good for short expirations[6]. Nevertheless the model has the virtue of having an exact expression for the implied volatility smile in the short-expiration limit $T \to 0$.

What alternative solutions exist for this problem?

2. THE TRIVIAL SOLUTION

A straightforward solution is to take Beta=0 in the definition of the SABR model. The resulting model is the stochastic volatility normal (or Bachelier) model. However, one expects that the results obtained with this model to be rather bad, not calibrating properly the smile.

3. THE SHIFTED SABR MODEL

The easiest[7] way to deal with low / negative rates in the SABR framework is to apply the SABR methodology to a shifted version of the forward rate:

$$dF = \alpha(F-S)^{\beta} dW_1, \qquad F(0) = f - S$$
$$d\alpha = \nu\alpha dW_2, \qquad \alpha(0) = \alpha_0$$
$$dW_1 dW_2 = \rho dt.$$

The implied volatility is in this case easily obtained from the original SABR implied volatility[8].

The drawback[9] of this idea is that the shift S (which appears here as a floor for the rate values) has to be somehow chosen or calibrated. In other words, one has to "guess how low the rates will go". If S is not chosen sufficiently large, a reassessment is necessary, meaning that all the computations have to be re-performed and valuations of option books can as a result be materially affected.

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6 Jim Gatheral "The Volatility Surface, A Practitioner's Guide", ISBN: 978-0-471-79251-2, Wiley Finance, p. 91. (See also Brigo-Mercurio, table p. 514).

7 As presented in the FB SABR paper (Antonov et al "The free boundary SABR: Natural Extension to Negative Rates"), p. 1

8 See for example Øyvind Grande Hansen "Multifactor Interest Rate Models in Low rate environments" (June 2013 - Norwegian University of Science and Technology) on p. 23, available on http://www.diva-portal.org/smash/get/diva2:650400/FULLTEXT01.pdf

9 This argument is given in the FB SABR paper p. 1

4. UNDERSTANDING ON STANDARD MARKET PRACTICES

On a no name basis we have contacted several interest rates derivatives traders from major European investment banks (mainly in London, Paris, Amsterdam and Brussels) to understand how they cope with the issue.

While most indicated no fully convincing solutions had been found at this stage (and some mentioned the Numerix article as an interesting way to explore to improve the solution) several of the traders mentioned that the "Shifted" solution should not be overlooked.

According to those traders the advantage hereof is that it enables to keep intact the dynamics of the SABR itself (which is sometimes seen as a non-straight-forward model to actively trade on). Of course the minimum rate has to be intuitively guessed but it probably one of the easier parameter to provide a conservative expert judgement on compared to other underlying assumptions (less controllable) used in alternative models variations.

5. THE FBSABR MODEL

a. Model

In their article[10], A. Antonov, M. Konikov and M. Spector present a generalization of the SABR model, based on a generalization of the CEV[11] model $dF_f = F_t^{\beta} dW$ (extending the corresponding density function support beside 0, so that the probability of obtaining negative values is > 0). The proposed approach roughly consists in summing two probability densities corresponding to solutions to the Fokker-Planck ("FP") equation[12] with "opposite" signs.

As one of those two densities is quasi singular in 0 (the corresponding CEV density is shaped like, say for instance $f^{-0,5}$), so is the FBSABR density. One can therefore observe some spike around 0 in the density:

[10] "The Free Boundary SABR: Natural Extension to Negative Rates" by Alexandre Antonov, Michael Konikov and Michael Spector at Numerix published on January 28, 2015 and available on http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2557046

[11] CEV = Constant elasticity of variance model

[12] More precisely, the authors look at the two FP equation's solutions, which behave differently around 0: an absorbing solution ($p_A \sim f^{1-2\beta}$) which is nice around 0 (for beta between 0 and 0.5), while a reflecting solution ($p_R \sim f^{-\beta}$) explodes around 0. As the FP equation is linear, a linear combination of the two solutions is a solution, so that

$$p(t,f) = \frac{1}{2}\left(p_R(t,|f|) + \text{sign}(f)\; p_A(t,|f|)\right)$$

is a solution. This is the Free Boundary CEV ("FBCEV") density.

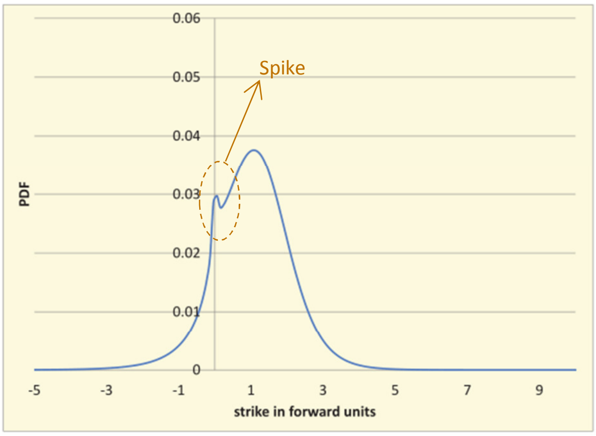

Figure 5: SABR model PDF for $T = 3Y$, $\beta = 0.1$.

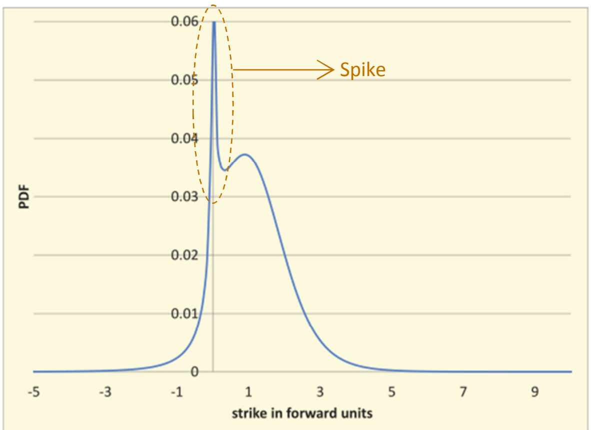

Figure 6: SABR model PDF for $T = 3Y$, $\beta = 0.25$.

Some traders we discussed with considered such spike in distribution as rather counter-intuitive. Our experience in also suggests that this could impact the stability of the model results in some cases. Especially when small differences in the assessment of an instrument's value can materially affect the result considered (e.g. when assessing sensitivities such as a derivatives Greek and specifically for second order Greeks such Gamma[13], Vanna[14] or Vol-Gamma[15])

We thus believe a **first point of attention** to consider when assessing or validation such modeling approach is to elaborate further on t**he impact such spikes in the distribution could have with regard to derivatives pricing and the computation of Greeks**, specifically in the context of the recent low rates (& strikes – see above) environment.

13 Sensitivity of Delta to underlying rates changes
14 Sensitivity of Vega to underlying rates changes
15 Sensitivity of Vega to implied volatilities changes (also sometimes referred to as "Volga")

b. Option pricing with FBSABR

Antonov, Konikov and Spector show in their article that computing the price of a European call option with FBSABR model is equivalent to computing it with FBCEV model in the case where the correlation between the Brownian motion governing the forward rate process and the Brownian motion governing its volatility is equal to zero.

This simple case is used to build an approximation of the option price when the correlation is not zero. Monte Carlo projections are used by the authors (with the scheme described in next section) to show that this approximation is rather efficient.

c. The simulation scheme of the FBSABR

The Euler scheme does not work to simulate a FRSABR process.

For this reason, the authors present another simulation scheme. The idea is to transform the FBSABR process F into a free (i.e. taking positive and negative values) Bessel squared process X (using a change of variable which is invertible and takes into account the sign). The CDF[16] of such a process is known, and based on non-central chi-squared distribution.

It is thus possible to use the following scheme:

1. Simulate uniform
2. Invert the X-CDF
3. Invert the variable change to obtain F.

However, the inversion of the X-CDF is time-consuming.

As a result Antonov, Konikov and Spector further suggest a scheme inspired by the Quadratic-Exponential scheme developed by Andersen[17]. The main idea is to approximate X by a squared Gaussian r.v. when s^2/m^2 (where m and s are first and second moments of X) is not too large, and to use the X-CDF inversion when s^2/m^2 is large.

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[16] CDF = Cumulative Distribution Function

[17] Andersen ”Efficient Simulation of the Heston Stochastic Volatility Model”, p. 13 (see “Suggested additional relevant readings” section for full details)

d. Attention point: A new role of the beta parameter

In the standard SABR model, the beta parameter may have a less important role.

For instance, as Brigo-Mercurio[18] write, in some elementary cases, *"market implied volatilities can be fitted equally well by different choices of β ∈ (0,1]. Hagan, Kumar, Lesniewski and Woodward, therefore, suggest to determine β either by historical calibration or by a-priori choice based on personal taste."*

Another similar opinion is that of Fabrice Douglas Rouah[19] "*The β parameter is estimated first, and is not very important in the model because the choice of β does not greatly affect the shape of the volatility curve*."

However, in the FBSABR model adaptation considered here it seems to us that the **beta parameter could now have a materially more important role**, as it governs in some sense the severity of the probability density singularity.

One could thus face the **need of a more sophisticated calibration for β**.

We thus believe **another key point of attention** in the case of the proposed model (for model assessment, calibration and for validation purpose) will be to **ensure the adequate interpretation of β and the adequacy of its calibration.**

[18] p. 510

[19] In, "The SABR Model", chapter 3 "Estimating Parameters", July 2011, Fabrice Douglas Rouah available on http://www.frouah.com/finance%20notes/The%20SABR%20Model.pdf

6. OTHER KEY POINTS OF ATTENTIONS FOR MODEL ASSESSMENT, CALIBRATION AND VALIDATION

We believe that from a theoretical point of view the content of the article by Antonov, Konikov and Spector is sound and achieves its goal. It is furthermore a robust innovative approach in a field where only few convincing solutions already pre-exist.

It is thus indeed a method worth consideration for market professionals active in interest rates derivatives. However some aspects of its practical implementation may not prove as straight-forward as one could initially expect it.

As elaborated above, we believe that both the presence of spikes in the obtained distribution and the role the Beta parameter deserve further investigation.

Further, with regard to the model validation aspects we would recommend to also consider the following points of attention:

- Overall the original FBSABR article suggests several numerical approximation methods which deserve specific assumption:
    - The accuracy of the analytical approximation for the non-zero correlation case is assessed by comparing its solution to a Monte Carlo scheme. We believe that the Monte-Carlo scheme comparison presented is sound and adequate, as it seems commonly used to evaluate SABR models and other stochastic volatility models (e.g. commonly used to assess the accuracy of mean-reverting models such as the Heston model). However, this Monte Carlos scheme in itself is by definition not exact. Before considering putting such model in production, we advise to more precisely **investigate how the numerical errors of the simulation scheme impact the quality of the results**. This is particularly important if derivatives Greeks (and especially second order Greeks such as Gamma[20], Vanna[21] or VolGamma[22]) have to be estimated.
    - Antonov, Konikov and Spector present some numerical results for a set of 'typical' values. We believe it is somewhat too light to support the real-life use of the model for actual trading purposes. We advise to **investigate to what extend the model produces stable result under less typical and extreme configurations**.

[20] Sensitivity of Delta to underlying rates changes
[21] Sensitivity of Vega to underlying rates changes
[22] Sensitivity of Vega to implied volatilities changes (also sometimes referred to as "Volga")

- The article does not present any evidence that the model can be calibrated to the observable market quotes in a fast and reliable manner. It should be investigated **what is the quality of the fit and how stable the calibrated parameters are over time**. This is specifically important for the purpose of dynamic hedging;
- More generally, the article does not present any result on the behavior of the Greeks. It should be investigated **how values sensitivities behave under the revised model dynamics and whether they are reliable**. Specific attention should be given to the behavior when approaching the zero singularity point.

Overall, **the calibration of the beta parameter would remain our main initial concern**. The calibration methodology generally typically used (as far as we know) to calibrate classical SABR parameters (i.e. an optimization over 3 [or 2] variable of the squared difference between observed and modelled volatilities) can prove materially more complex once and additional variable is added. If, as we suspect, the importance of beta's proves more significant than in classical SABR (but this as to be rigorously assessed, of course), we are not sure to see the real advantage of the FBSABR model over the shifted SABR (whose major problem was the calibration and eventual reassessment of the shift). In this case indeed one would introduce again a less intuitive parameter to calibrate. Hence, the risk could be that migrating from shifted SABR to FBSABR would only mean going from a calibration problem to another calibration problem (less prone to comprehensive expert judgement). On this point we need however to stress out the fact that this is just an intuition which deserves to be checked out in details.

We also believe an important (and possibly initial) step in a validation process will be to **implement the Monte Carlo scheme proposed in the paper** (and maybe the second one that the authors present as slow [possibly to be ran on somewhat more limited number of scenarios], as a check) in order to have a better understanding of the resulting distribution.

Furthermore we believe it could prove interesting to perform **a comparison with the distribution resulting from the shifted SABR model** (as this approach also seems to have been used so far by some peers in the market).

7. OTHER EXTENSIONS OF THE SABR MODEL

Other extensions of the SABR model have been developed, such as the DD SABR[23] and ZABR[24] models. However, none of them both solves the problem of low/negative rates.

8. REACFIN'S SUPPORT

Reacfin is a consulting firm specialized in Risk Management, Actuarial Science, Portfolio Modeling and Quantitative Finance. We regularly support financial institutions in the development, the implementation and the validation of their new models.

With this White Paper we aim at illustrating our structured approach to quantitative model assessments.

We deeply believe that risk taking & innovation are inherent to the business models of financial institutions yet only scrupulous & systematic approaches can ensure the adequacy and robust implementation of new models.

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Reacfin has recently worked for several large banks and trading houses in the development of their latest pricing models for derivatives, collateral management and portfolio management. We also performed a set of validation missions in similar fields of expertise.

Specifically issues related to the modeling of financial instruments in low/negative rates environment are topics on which through different recent assignments with could acquire a distinctive expertise.

We look forward having the opportunity to also serve your company soon.

In the following exhibits, we illustrate our focus and provide a few additional examples of our recent assignments.

[23] Kienitz-Wittke_Option Valuation in Multivariate SMM-SABR Models (with an Application to the CMS Spread)
[24] Andersen_ZABR - Expansions for the Masses

Reacfin
Know-How to Risk

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We develop **innovative solutions and robust tools** for Risk and Portfolio management.

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- Pricing of financial instruments & development of ALM models
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- Tools development (Valuation, Pricing, hedging, portfolio replication, etc.)
- Design of Capital Management solutions

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- Internal & regulatory reporting (KRI's & KPI's dashboards)
- Model Review frameworks
- Model Documentation

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- DFA* Models
- Capital Requirement assessment
- Business valuation support
- Product development (pricing, profitability,..) & Reserving
- Model validation

Non-Life

- Reserving: triangle methods, individual claims modelling
- Pricing: frequency and severity modelling, large claims analysis, credibility methods, commercial constraints
- DFA models: cash-flows projection, calibration of models
- Reinsurance: modelling covers, optimal reinsurance programs

(*) DFA = Dynamic Financial Analysis

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- Respecting the principle of proportionality
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No black box Solutions

- We deliver results, not reports!
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- Close cooperation with our clients

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- Lean & efficient tailored project management
- Regular progress reviews
- Close cooperation with our clients

Documentation, coaching & training

- Clear & comprehensive documentation compliant existing or upcoming regulation
- Adapted trainings at all levels of the organisation
- Coaching support for implementation and operationnalisation of processes

Example of previous assignment:

Complex financial instruments & derivatives pricers development

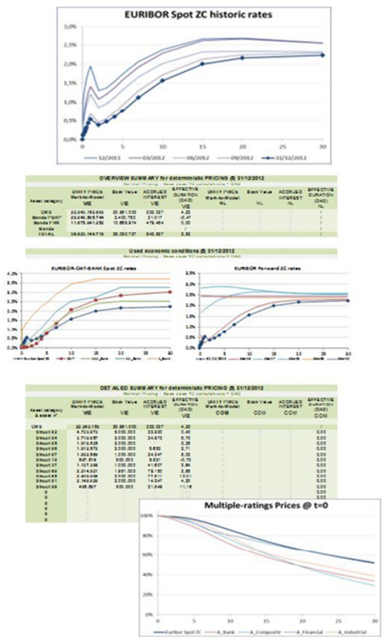
All dummy numbers & graphs for illustrative purposes only

Client Situation

- European Banks & insurance companies/groups
- Internal needs to price non liquid asset or highly structured products
- Internal need to challenge Mark-to-Market prices or price estimations coming from counterparties
- Requirement to estimate complex products price under stress testing (SOLVENCY II or other regulatory/internal purpose)
- Need to integrate complex product pricer into existing ALM model

Issues

- Insure market consistency
- Tool should be
    - Easy to understand/use
    - Auditable or validated
    - Operational
    - Integrated
- Addition of price sensitivity to economics indicators



Reacfin Contribution

- Propose collaboration for model conception and metrics definition
- Selection and classification of products
- Methodological choices for each products class
- Model structuration and implementation
- Model testing and operationalization (incl .doc.)
- Parameters calibration and pre-production tests
- Supporting client company in audit and external review
- Advisory in complex products management

Results & Benefits

- Improved product comprehension
- More transparency and knowledge for products management
- Multiple uses for business or regulatory needs
- More independence with counterparties
- Integration to internal financial reporting system
- Optimal ratio "Complex tool/costs"

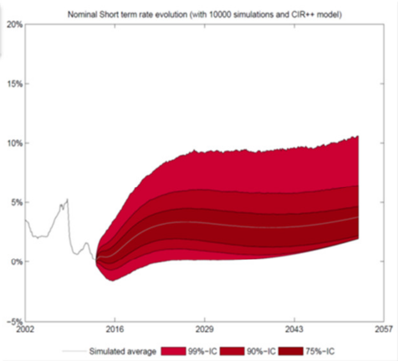
Example of previous assignment:

Economic Scenario Generator and applications

All dummy numbers & graphs for illustrative purposes only

Clients Situation

- Large liability driven financial services conglomerate in Europe: valuation of a quarterly reported complex asset/liability on the balance sheet
- Developing of a fully integrated ALM, CPM, collateral management and derivatives pricing model
- Valuation of derivatives, structured products and insurance contracts using Monte-Carlo techniques

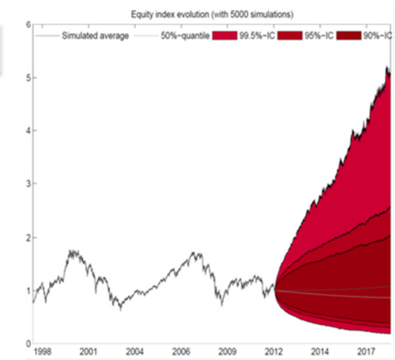


Reacfin Contribution

- Provide risk neutral or real world scenarios of main market risk drivers : interest rates, inflation, default and spreads, equity, real estate, …
- Use of Stochastic Advanced market model (Heston, LMM, JLT,…) and their interdependence.
- Complete documentation on the models: methodology, calibration, simulations and testing
- Incorporation of the scenario outputs in the structure of your model
- Possibility to provide a simulation tool rather than scenarios

Issues

- Provide market consistent and arbitrage-free pricing of asset/liability using Monte-Carlo techniques
- Provide potential future scenarios evolution of the economic and financial world for risk management purposes
- Provide sensitivities w.r.t. to several risk drivers



Results & Benefits

- Improved risks quantification & business considerations & Management
- Quantitative approach for dependences between risk drivers
- More accurate pricing of market products using innovative approaches other than the usual short rates models or the basic Black & Scholes model
- Expert idea of the potential future scenarios evolution of the economic and financial world (+ stress tests) and their probability of occurrence.

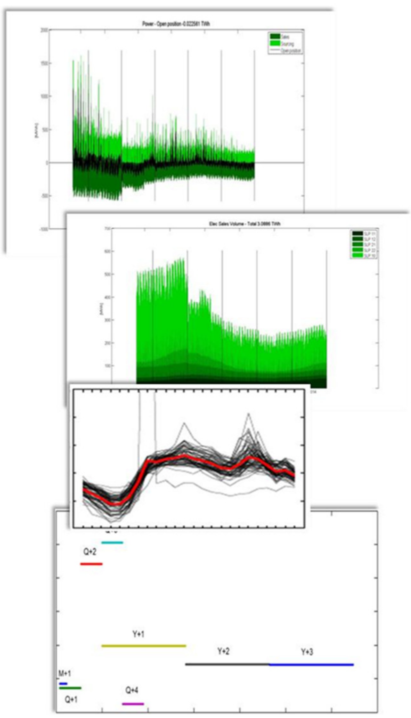
Example of previous assignment:

Energy risk management & modeling

All dummy numbers & graphs for illustrative purposes only

Client Situation

- Large energy (electricity and gas) producer and retailer
- Internal needs to a proper assessment of the market risk associated to the portfolio on a hourly basis
- Internal need to develop a hourly price forward curve for sourcing and sales pricing purposes
- Internal need to visualize hourly physical and financial open position
- Internal need to develop risk metrics and tolerance limits



Reacfin Contribution

- Development of a full methodology in order to build the hourly price forward curve for electricity and gas
- Computation of the open position (sales – sourcing) on a hourly basis taking into account hedge effects between portfolio's
- Development of risk metrics in order to assess market risk on the P&L : VaR, EaR, stress testing
- Close collaboration with client's teams to really capture all client's characteristics and to work on a complete transparence with an open box process
- Identification of the different seasonal effects inherent to the energy market and client's clients
- Training of the associated teams to use and, extend and improve the tool by their own

Issues

- Data quality and availability
- Lack of comparable benchmarks and understanding of sound market practices

Results & Benefits

- Improved risks & business comprehension
- Globalization of the risk assessment
- Creation of a market pricing tool
- Ability to extend the model by client's teams if business develops

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9. SUGGESTED ADDITIONAL RELEVANT READINGS

On top of the article's bibliography, we could suggest the following additional relevant reading (in chronological order of publication) on related topics:

- Graeme West (Dec. 2005), "Calibration of the SABR Model in Illiquid Markets", Journal of Applied Mathematical Finance, Vol. 12, No. 4, 371–385
- Leif Andersen (Dec. 2006), "Efficient Simulation of the Heston Stochastic Volatility Model" (Banc of America Securities) available on http://www.ressources-actuarielles.net/EXT/ISFA/1226.nsf/0/1826b88b152e65a7c12574b000347c74/$FILE/LeifAndersenHeston.pdf
- Louis Zhuang (March 2010), "Using LMM-SABR Model to Manage Smile Risk of Fixed Income Portfolio" (London School of Economics) available on http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1566508
- Jörg Kienitz and Manuel Wittkey (June 2010), "Option Valuation in Multivariate SMM/SABR Models - with an Application to the CMS Spread" (University of Bonn and Deutsche Postbank AG) available on http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1469554
- Jesper Andreasen and Brian Huge (Dec. 2011), "ZABR -- Expansions for the Masses" (Danske Markets, Copenhagen) available on http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1980726
- Bin Chen and Cornelis W. Oosterlee (January 2011), "Efficient unbiased simulation scheme for the SABR stochastic volatility model" (TU Delft) available on http://ta.twi.tudelft.nl/mf/users/oosterle/oosterlee/SABRMC.pdf
- Richard Jordan and Charles Tier (May 2011), "Asymptotic approximations to CEV and SABR models" (Illinois Institute of Technology), available on http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1850709
- Jason Vinar (January 2012), "Using the SABR Model" (Presentation by Ameriprise) available on http://www.math.umn.edu/finmath/modeling/Materials/Vinar_presentation_Jan2012.pdf
- Alexander Antonov and Michael Spector (March 2012), "Advanced analytics for the SABR model" (Numerix Quantitative Research) available on http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2026350
- Paul Doust (March 2012), "No-arbitrage SABR" (Royal Bank of Scotland), The Journal of Computational Finance (3–31) Volume 15 / Number 3
- Tran H. Nguyen and Anton Weigardh (March 2014), "The SABR Model - Calibrated for Swaption's Volatility Smile" (Mälardalens Högskola) available on http://mdh.diva-portal.org/smash/get/diva2:704410/FULLTEXT01.pdf

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