A Unified Approach for the Modeling of Rating Factors in Workers’ Compensation Insurance
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Abstract

Workers’ Compensation is an insurance cover whereby an employer compensates for the lost wages and medical expenses of an employee who is injured on the job. As with other types of insurance, premium rates for workers’ compensation policies are calculated according to several different factors. In Belgium, current market practice consists of pricing workers’ compensation based on a hierarchical credibility model, i.e. the Jewell model. Hierarchical credibility models provide a nice hierarchical procedure in the calculation of premiums. First, we produce an expected aggregate claim amount for the whole line of business. Then, we distribute this amount over lower levels (top down approach).

In this paper, we define a new tariff structure for workers’ compensation by combining hierarchical credibility models with a GLMM approach in the final pure premium model. We start with the calculation of a basic pure premium rate in function of some explanatory variables by using a Generalized Linear Mixed Model (which is an extension of the well-known Generalized Linear Models). The advantage of such a model is that we can now fit ordinary rating factors and hierarchical rating factors at the same time. Afterwards, this basic premium rate is adapted according to the client’s history of claims data with the use of the Bühlmann-Straub credibility model.
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1. Introduction

Workers’ Compensation is an insurance cover whereby an employer compensates for the lost wages and medical expenses of an employee who is injured in the course of employment. It is designed to cover an Occupational Accident, which means an accident occurred at work, on the journey between the employee’s residence and the place of work, or between the workplace and the place where the employee takes a meal. Compensations include medical, surgical, pharmaceutical costs and hospital care as well as equipment. Furthermore, temporary allowances and annuities once the incapacity percentage is known are also part of the cover. For most of the countries, this form of insurance is part of the social security. In Belgium, this activity is provided by the private insurance sector. Most accidents are indemnified by insurance companies, but in some cases the loss is compensated by the federal agency Fediris (a merge between the Fund for Occupational Accidents and the Fund for Occupational Diseases).

Tariff or premium structures for workers’ compensation are usually characterized by the degree of risk sharing among employers which belong to the same risk category. To create an incentive for employers to reduce their total incurred claim amount, most systems are, to a certain extent, experience rated, which means that the contribution of an individual firm is an increasing function of the cost and/or number of claims it generates for the system. However, in order to avoid extreme penalties on smaller firms which suffered severe workplace accidents in a certain year, some systems also apply a degree of experience rating which is increasing with the size of the firm. In this way, smaller firms can mutualize their risk.

In this paper, we define a new tariff structure for workers’ compensation by combining hierarchical credibility models and a GLMM approach in the final pure premium model. We start from a basic pure premium rate which is then adapted according to client’s history of claims data. The advantage of this new model is that we can fit ordinary rating factors and hierarchical rating factors at the same time. Therefore, we use a Generalized Linear Mixed Model (which is an extension of the well-known Generalized Linear Models). The remainder of the paper is structured as follows. In Section 2, we specify all the separate elements that are needed in order to build the new tariff structure, i.e. credibility models, the GLM/GLMM and the Tweedie distribution. Application of those methodologies on a claims database leads us then to the modeling of the new tariff structure which is outlined in Section 3. Section 4 concludes the paper.
2. Approach

In this section, we discuss in detail a new approach to obtain a rating structure for Workers’ Compensation. We start with a subsection on data input and pre-processing. Afterwards, we outline the precise methodology and we end the section with some adjustments that can be made for commercial reasons.

2.1. Data input and pre-processing

In order to investigate different modeling techniques for a new tariff structure, specific data will be required. We should therefore make sure the required data are available and of good quality before starting the model development. We assume that we have a complete claims database available over a certain period. Claims data accuracy being an important basis of any rating structure model, this step should not be neglected especially in a very long tail line of business like workmen compensation. For each claim in the database, we have at our disposal several characteristics related to the claim. Examples of those can be the following: the NACE code of the company, the salary, the total amount of payments and reserves, the accident year or the postcode.

2.1.1. Calculation of Best Estimate reserves

Due to the specific character of Workers’ Compensation in Belgium, insurers have to build up Be-GAAP annuities reserves according to specific legal parameters, such as the interest rate or mortality tables. Furthermore, the set of legal parameters depend on the accident date and specific rules apply when the degree of the employee’s injury reaches pre-defined levels. The pricing process has to be calibrated on a best estimate view of ultimate future claims cash flows (in order to avoid over- or underestimation of the total payments). In case, only Belgian GAAP accounting values are available at contract level, it will be necessary to apply a methodology to transform the legal view technical provisions into best estimate view. The best approach here would be to perform such a conversion of a claim per claim basis. However, depending on the granularity level of the available best estimate cash flow data, this is not always possible. Instead, we can produce conversion factors for different reserve categories besides the accident year, such as a distinction between way to work and occupational risk, or the categories workers and employees.

If discounted reserves are used in the computation of conversion factors (e.g. by discounting the future yearly cash flow amounts by the EIOPA annual zero-coupon risk-free spot rate curve without volatility adjustments), application of these conversion factors on the yearly claim per claims legal reserves will produce estimation of discounted Best Estimate reserves. Remark that if it is expected that the investment return is higher than the risk-free curve of EIOPA, then the rates which will come out of the technical model later on should be adjusted with an extra discount correction factor.

2.1.2. Large claims treatment

In traditional actuarial reserving, large claims are often separated from other claims (i.e. “attritional” claims) for analysis. Having a limited number of large claims might mean there is not enough data on which to base a separate allowance for large claims, but on the other hand not excluding them can mean that an “all claims” analysis shows unstable experience due to distortion from those few large claims. A difficult problem consists in selecting an adequate threshold to separate attritional claims from large claims. Some techniques that are used to help in the determination of the threshold are the following: the mean excess function, the Gertensgarbe plot and the index plot. In order to select the threshold, the actuary will need to exercise some expert judgement by investigating the outcomes of those methods and put in perspective with the reinsurance (excess-of-loss) program.

The pure premium model is usually fitted on the attritional claims so that the total claims amount will only be an estimation of the total amount for the claims which are capped to the threshold value. In order to take into account the full exposure, a possibility is to apply a loading calculated as the ratio between the total observed claims amount and the total claims amount estimated by the pure premium model. Another
possibility is to model the large claims separately (e.g. by fitting a Generalized Pareto Distribution on the excess over the threshold combined with a model for the frequency of large claims).

2.2. Methodology

In order to obtain a rating structure that allows for extrapolating into the future, a given population has to be divided into homogeneous classes or cells. Having homogeneous groups enables one to gain precision and also allows anticipating the future trends. Such a classification of the portfolio could be achieved using different techniques such as Generalized Linear Models (GLM), Machine Learning or Hierarchical Credibility Models. In this paper, we will discuss both Hierarchical Credibility Models and the GLM approach. The main idea would be to combine results of both techniques in the final pure premium model.

2.2.1. Credibility models

Credibility theory studies how to incorporate the information that is cell-specific with the information gathered for the whole portfolio premium calculation. For the insurance industry, equilibrium between different classes of risk can be achieved through credibility, allowing the company to react more swiftly to competition by recognizing individual and group characteristics. Originally, the general credibility formula was used in the field of Workers’ Compensation insurance to calculate the a posteriori premium, i.e.

\[ C = (1 - Z)A + ZB \]

where \( A \) is the a priori premium coming from the model, \( B \) is the contract observed average mean computed from the past loss history and \( C \) is the compromise of the premium. The value \( Z \) is the weight given to the past loss history and expresses how ‘credible’ the individual contract experience is, such that \( 0 \leq Z \leq 1 \). If \( Z \) is equal to zero, the a posteriori premium is entirely based on the model and not influenced by the contract past observations. However, in case the volume of the past history is sufficient, the credibility factor \( Z \) is closer to 1 and there is little need to use the entire a priori premium because the individual class is credible enough.

Current market practice in Belgium consists of pricing workmen compensation according to the Jewell credibility model. This model provides a nice hierarchical procedure in the calculation of premiums. First, we ascertain an expected aggregate claim amount for the whole line of business. Then, we successively distribute this amount over lower levels (top down approach). In the Jewell Credibility model, three levels of granularity are used:

1. Lob level (global portfolio level)
2. Risk classes: NACE-codes with similar combination of risk factors
3. Tariff groups: tariff groups with similar risk profile

This structure was designed some years ago by Assuralia and is illustrated in Figure 1. Remark that depending on data availability more than three levels can be used inside the model.
2.2.2. Generalized Linear Models (GLM)

Generalized linear models have by now become a standard approach used for non-life pricing in many countries. GLM is a rich class of statistical methods, which generalizes the ordinary linear models in two directions. First of all, the mean and covariates are linked by the so-called link function which can be more general than a linear function. Secondly, the response variable can be modeled by a general class of distributions besides the normal distribution.

Based on those elements, a transformation of the mean response is decomposed into a sum of risk factor effects (or the so-called score). We have

$$E[Y_i] = \mu_i \text{ with } g(\mu_i) = \text{score}_i.$$  

Using GLM, we can explain the premium rate (to be applied to the total salaries’ amount) in function of some explanatory variables (e.g. category of payroll amounts, risk factor of the activity, classes, location, number of employees, …). As this is a regression technique, we will obtain a tariff even for segments where few data is available, supplemented with confidence intervals. The final structure of the tariff using such an approach will be strongly dependent on the amount of risk factors available in the database.

2.2.3. Tweedie GLMs

The compound Poisson distribution with Gamma claim sizes is a very common model for premium estimation insurance. Under this distributional assumption, the claim frequency follows a Poisson distribution while the claim severity follows a Gamma distribution with independence between both. However, when a separate analysis for claim frequencies and severities is not needed or if only annual claims data is available, a Tweedie distribution can be used to estimate the mean aggregate loss using GLMs directly. Furthermore, it enables us to better model data with a positive probability at zero and a continuous distribution otherwise.

A Tweedie GLM is a generalized linear model where the response variable now follows a Tweedie distribution, i.e. $Y \sim \text{Tweedie}(\mu, \phi, q)$. A Tweedie distribution is a family of probability distributions which possesses the property that its variance relates to its mean by a power law, i.e. $V[Y] = \phi \mu^q$ for some positive constant $q$. By assuming that $1 < q < 2$, we arrive at a compound Poisson-Gamma distribution. In this case, it is recommended to use such a distribution instead of a Poisson $(q = 1)$ or Gamma $(q = 2)$ distribution since no specific analysis is needed for frequency and severity and we only have a few explanatory variables at our disposal.
Finally, we assume a log link function in the approach to link the response variable with the mean, such that \( \mu = \exp(\beta^T X) \). Similarly, we can write

\[
\mu_i = \exp(\beta_0) \cdot \exp(\beta_1 X_{1i}) \cdot \exp(\beta_2 X_{i2}) \cdot ...
\]
\[
\mu_i = \gamma_0 \cdot \gamma_{1i} \cdot \gamma_{i2} \cdot ...
\]
\[
\mu_i = \gamma_0 \cdot \gamma_{i1},
\]

where \( \gamma_0 \) is the value of the reference class and \( \gamma_{1i} \) is the product of the relativities for contract \( i \). Previous formula shows the multiplicative character of the relativities which is the reason why we choose a log link function.

2.2.4. Generalized Linear Mixed Models (GLMM)

Consider now a categorical covariate with many levels, such as a car model classification in motor insurance or the sector of a company (NACE code) in workers’ compensation insurance, with many data for some of the levels and limited data for others. In that case, a credibility model and a Tweedie GLM can easily be combined into a mixed model with the familiar multiplicative mean structure. This resulting structure is then called a Generalized Linear Mixed Model or GLMM. Basically, generalized linear mixed models are an extension of generalized linear models as they include both fixed and random effects (hence a so-called mixed model). If we again assume a log link function, we can now write

\[
E[Y_{ij}|U_j] = \gamma_0 \cdot \gamma_{1i} \cdot U_j \quad \text{with} \quad E[U_j] = 1.
\]

The additional random effects \( U_j \) are all independent and identically distributed random variables and are also independent of the response variable. In previous formula, the variable \( Y_{ij} \) corresponds to the contract \( i \) belonging to a certain sector \( j \). Remark that each sector can also contain several subsectors corresponding to a specific hierarchical structure. In that case, we add two additional random effects \( U_j \) and \( U_{jk} \).

Knowing the factors \( \gamma_0 \) and \( \gamma_{1i} \), we can estimate the effects \( U_j \) and \( U_{jk} \) by using the Bühlmann-Straub credibility model. An iterative procedure can therefore be put in place in order to estimate the Tweedie GLMM regression parameters in the presence of random effects: this is the GLMC algorithm of Ohlsson (2008) which iterates GLM Tweedie fits and Bühlmann-Straub predictions until convergence.

3. A new technical tariff structure

Now that we explained all separate elements of the methodology, we can build up a new tariff structure for Workers’ Compensation by combining hierarchical credibility models and a GLM approach in the final pure premium model. We start from a basic pure premium rate which is then adapted according to client specific data. The following techniques are considered:

- Generalized Linear Mixed Model (GLMM)
- Experience rating using Bühlmann-Straub credibility model

3.1. Basic pure premium rates

In this subsection, we discuss the Generalized Linear Mixed Model which leads us to a basic premium rate for each class of the rating system that is used in the model. The a priori ratemaking model, which models the annual total claims amount, combines the following techniques for different kind of rating factors:

- For ordinary rating factors, we use the Tweedie Generalized Linear Model (GLM). Examples:
  - Risk level: A possible rating factor that depends on the activity of the company can be found on the website for Prevention and Interim. Companies are here classified into groups according to the risk associated to the activity. A possible classification in
groups (with corresponding NACE codes) consists of the 4 risk levels: Low, Medium, High and Very High.

- Salary: In order to add salary as a possible rating factor in the tariff structure model, we can create different wage classes to obtain a categorical variable. The specific pattern will depend on the available dataset and can be calculated based for example on quantiles of the available range of salaries or such that the total claim experience in each wage class is the same.

- Region/Zone: As the region can have an influence on the level of premium paid by the company, we merge postcodes into specific regions based on the underlying pattern that is left after including all the other rating factors. The different regions that we obtain in this case, can be used as an extra rating factor within the calculation of the basic premium rate. More information on this procedure can be found in the appendix 1.

- For multilevel hierarchical rating factors, we use the Tweedie Generalized Linear Mixed Model (GLMM). Examples:
  - NACE code: The NACE code system is the European standard for industry classifications and consists of multiple levels. It classifies companies according to their economic activities into different classes and groups. A classification developed by Assuralia in 2008 consists of the following NACE structure:
    - NACE Class: 30 classes which are merged according to a similar risk profile.
    - NACE Group: 294 groups consisting of NACE codes with the same tariff (if no other rating factors would be taken into account).

Taking into account the chosen rating factors, the pure premium rate \( P \) is then calculated as the product of the reference class premium rate \( P_0 \) and the relativities \( R \) of the corresponding rating factors. In the most general case, we have \( P = P_0 \cdot R \). Because there is a hierarchical rating factor involved, the specific values of the relativities can be found by fitting a GLMM. After a thorough analysis of the model fit, some rating factors can possibly be removed in the calculation of the basic pure premium rate.

### 3.2. Client specific pure premium rates (with credibility model)

In previous section, we calculated the a priori technical price for each contract by using the GLM/GLMM model. This a priori price is simply the product of the reference rate and the relativities corresponding to the risk characteristics. In this way, all contracts with the same characteristics will get the same a priori price. In this section, we use the credibility model of Bühlmann-Straub (see appendix 2) to calculate the a posteriori premium, which means we refine the a priori premium for each contract using the contract specific loss history. The a posteriori premium of each contract becomes a weighted mean between:

- The a priori premium rate coming from the GLMM/GLM model.
- The contract observed average mean computed from the contract past loss history.

The weighted mean is based on the credibility weight \( \alpha_i \) computed from the Bühlmann-Straub model. This credibility factor \( \alpha_i \), which is between 0 and 1, can be seen as the weight given to the contract observed average mean. The credibility factor is different for each contract. Under the Bühlmann-Straub model, it is computed as follows:

\[
\alpha_i = \frac{M^2 \bar{w}_i}{\Sigma^2 + M^2 \bar{w}_i},
\]

where \( \Sigma^2 \) and \( M^2 \) are specific parameters computed at portfolio level and \( \bar{w}_i \), can be seen as the sum of the "normalized" salary over all accident years depending on the contract salary amount \( w_i \) and the a priori premium \( \mu_i \) which is specific for each contract. The bigger \( \alpha_i \) is, the more the final premium will be influenced by the contract past loss history. On the contrary when \( \alpha_i \) is close to 0, the a posteriori premium is nearly equal to the a priori premium coming from the GLM/GLMM.
Contracts with high salary amount and large past losses will therefore be penalized as those cases will receive a higher $\alpha_i$, which means that the final premium will be mainly influenced by the contract past observations.

3.3. Commercial premium rate

Previous subsections only elaborate on the pure technical price which is the amount the insurance company should charge in order to be able to indemnify all the claims, without loss or profit. In a final step, a commercial premium rating structure should be developed in order to commercialize the product to clients. The pure premium rate is charged such that the commercial premium also covers the administration costs and commissions. The final commercial premiums often differ from this loaded pure premium tariff (technico-commercial premium) because of:

- Positioning constraints (e.g. different tariff for portfolio and new entrants);
- Commercial constraints (e.g. positioning with respect to the pricing of the competitors, potential radical change for some risk categories);
- IT constraints (e.g. variables of the technical tariff not available in the IT pricing tool, IT limitations in the structure of the commercial prices);
- Legal constraints (e.g. some variables cannot be used in the final ratemaking).

Therefore some adjustments are performed to the technico-commercial premium in order to take into account all these constraints and arrive at the final commercial tariff, which covers the claim amount as well as the commission and administration costs.

4. Conclusion

In this paper, we proposed a new tariff structure for workers’ compensation by combining a hierarchical credibility model with a GLMM approach. To calculate the final pure premium, we started with the estimation of the basic pure premium rate by using a Generalized Linear Mixed Model. Such a model can be seen as a unified approach in the modeling of rating factors, as it can fit ordinary rating factors and hierarchical rating factors at the same time which is not possible with the classical Jewell model usually used on the market. In a final step, an experience rating system is included based on the Bühlmann-Straub credibility model, to let the premium of the individual company evolve, at least partially, in function of its own history of claims data.

This model therefore allows to take into account in an unified approach the 3 main types of rating factors that are available to incorporate in a workers’ compensation tariff structure: the hierarchical classification of the activity of the company (based on the NACE codes), the available a priori rating factors (wages amount, number of employees, zone,….) and the specific claims history of the company.
5. Bibliography


6. Reacfin’s support

Reacfin is a consulting firm specialized in Risk Management, Actuarial Science, Portfolio Modeling and Quantitative Finance. We regularly support financial institutions in the development, the implementation and the validation of their new models.

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(*) DFA = Dynamic Financial Analysis
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- Risk appetite framework definition
- Overall design of Risk management framework
- ORSA organization and process
- Validation of internal model qualitative aspects
- Dependency modeling and testing
8. Appendix

Appendix 1: Region mapping analysis

As the region can have an influence on the level of premium paid by the company, we can adopt a risk classification according to the geographical zone where the policyholder lives (urban/non-urban for instance, or a more accurate splitting of the country according to postcodes).

The aim is to introduce in the tariff a new explanatory variable based on the policyholder’s postcode, while zones or regions (groups of different postcodes) have to be constructed and their relativities have to be established. To determine the zones similar in terms of risk, we first aggregate the total claims by district. We can then calculate the ratios between the observed total claims and the sum of the salary for each district (normalized claims). These ratios can be interpreted as “residuals” by district. The idea is then to structure these residuals in order to define a categorical variable that will improve the risk prediction of the model.

Sorting these ratios to create classes for the new variable, can lead to zones and corresponding relativities (thanks to an additional GLM analysis on the new categorical variable) which are not very smooth and difficult to explain from a commercial point of view. In such a situation, we can try to smooth the result by adding the coordinates \((u, v)\) of the district as a continuous explanatory variable. We then fit the following GAM model

\[
\frac{\text{TotalClaims}_j}{\text{SAL}_j} \sim \text{Tweedie} \left( \exp \left( f(u_j, v_j) \right) \right).
\]

The function \(f\) appearing in the latter formula is left unspecified but assumed to be smooth. Based on this assumption, it can be estimated with local regression models of GLM-type called Generalized Additive Models (GAMs). The obtained results are smooth by construction with this methodology. In the following figure, we plot a dummy example of a Belgian map obtained after the smoothing method, where the zones are built such that the exposure (salary) is the same in each zone.

![Smoothed GAM geographical function](image)

**Figure 2:** Example of a Belgian map obtained after the GAM smoothing method (where each zone has the same exposure).
Appendix 2: The Bühlmann-Straub credibility model

In this appendix, we give more details on the Bühlmann-Straub credibility model that is used to take into account the client specific history of claims data. Consider a given policy $i$ observed during accident years $t = 1,\ldots,T$. This policy is represented by a random vector $(\Lambda_i, Y_{i1}, Y_{i2}, \ldots, Y_{iT})$ where

- $Y_{it}$ is the “normalized” claim amount for policy $i$ in year $t$, i.e. total claims divided by salary $w_{it}$;
- $\Lambda_i$ is an unknown risk factor.

Given $\Lambda_i$, the random variables $Y_{i1}, Y_{i2}, \ldots, Y_{iT}$ are mutually independent. At portfolio level, the random variables $\Lambda_1, \Lambda_2, \ldots, \Lambda_n$ are iid with $E[\Lambda_i] = 1$, and the random vectors $(\Lambda_i, Y_{i1}, Y_{i2}, \ldots, Y_{iT})$, for $i = 1,2,\ldots,n$, are mutually independent.

Let

$$\bar{w}_{it} = \sum_{t=1}^{T} w_{it} = \sum_{t=1}^{T} w_{it} \mu_i^{2-q}$$

be the total volume for policy $i$ and define

$$\bar{y}_{it} = \frac{Y_{it}}{\mu_i}.$$ 

Then the credibility factor for policy $i$ is given by

$$\alpha_i = \frac{M^2 \bar{w}_{it}}{\Sigma + M^2 \bar{w}_{it}},$$

where $M^2 = V[\Lambda_i]$ and $\Sigma^2 = E[\phi(\Lambda_i)^q]$. The variables $Y_{it}$ are assumed to be conditionally Tweedie-distributed such that the value $q$ involved in the formulas is the Tweedie parameter. The credibility estimator is given by

$$\hat{\Lambda}_i = 1 - \alpha_i + \alpha_i \frac{1}{\bar{w}_{it}} \sum_{t=1}^{T} \bar{w}_{it} \bar{y}_{it}.$$ 

To estimate the parameter $\Sigma^2$, we use the formula

$$\hat{\Sigma}^2 = \frac{1}{n(T-1)} \sum_{i=1}^{n} \sum_{t=1}^{T} \bar{w}_{it} \left( \frac{\bar{y}_{it}}{\bar{w}_{it}} - \frac{1}{\bar{w}_{it}} \sum_{t=1}^{T} \bar{w}_{it} \bar{y}_{it} \right)^2.$$ 

Similarly, the estimation of parameter $M^2$ follows from

$$\hat{M}^2 = \frac{\bar{w}_{it}^* - \Sigma \hat{\Sigma}^2}{\bar{w}_{it}^*} (S^2 - (n - 1)\hat{\Sigma}^2),$$

where

$$\bar{w}_{it}^* = \sum_{i=1}^{n} \sum_{t=1}^{T} \bar{w}_{it}$$

and

$$S^2 = \sum_{i=1}^{n} \bar{w}_{it} \left( \frac{1}{\bar{w}_{it}} \sum_{t=1}^{T} \bar{w}_{it} \bar{y}_{it} - \frac{1}{\bar{w}_{it}} \sum_{t=1}^{T} \bar{w}_{it} \bar{y}_{it} \right)^2.$$
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A Reacfin White Paper on Workers’ Compensation:
A Unified Approach for the Modeling of Rating Factors in Workers’ Compensation Insurance

by Ben Stassen, Michel Denuit, Samuel Mahy, Xavier Maréchal and Julien Trufin

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