Abstract

In most industrialized countries, including Belgium, mortality at adult and old ages reveal decreasing annual death probabilities. The calculation of expected present values (for pricing or reserving) for long-term survival insurance benefits thus requires an appropriate mortality projection in order to avoid underestimation of future costs. The so-called projected life tables include a forecast of the future trends of mortality. This paper updates previous projections based on recent mortality statistics, for the general population and for the insurance market. A discussion of the strategies to deal with the systematic longevity risk concludes the study.
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1 Introduction and motivation

In this paper, we update previous mortality projections using the user-friendly tool Thanatos produced by Reacfin, a UCL spin-off based in Louvain-la-Neuve. The present study concentrates on the period 1950-2015. For an analysis of mortality trends over a longer period, we refer the interested reader to Chapter 3 of Pitacco et al. (2009) or to Brouhns and Denuit (2002a). For a general account of methods for projecting mortality, see Delwarde and Denuit (2005) and Pitacco et al. (2009). Of course, this is not the first projection of mortality for Belgium. The Belgian Institute of Actuaries IABE (former KVBA-ARAB) launched a working group involving practitioners and academics in the early 2000s, chaired by Philippe Delfosse. This active group produced the first projected life tables for the Belgian insurance market, that were presented and distributed during a colloquium that took place in Brussels. The methodology behind these projections has been summarized in Brouhns and Denuit (2002b,c). More recently, as a follow-up to the 2002 mortality study, Antonio et al. (2015) have produced updated mortality projections for IABE, based on an original methodology involving a group of countries. The model is an application of the AG 2014 projection methodology of the Dutch Koninklijk Actuarieel Genootschap to the Belgian setting.

Before embarking on a mortality projection case study, we have to decide about the type of mortality statistics that will be used. Using market data allows us to take adverse selection into account. However, basing mortality projections on market data implicitly would mean that no structural breaks have occurred because of changes to the character of the market, or modifications in the tax system or in the level of adverse selection, for instance. Assume, for example, that the government starts offering incentives to individuals from the lower socio-economic classes to buy life annuities in order to supplement public pensions. Using market data would result in a worsening in mortality because of a modification in the profile of the insured lives (as lower socio-economic classes usually experience higher mortality rates). Hence, this will artificially modify the mortality trends for the market. It is, thus, impossible to separate long-term mortality trends from modifications in the structure of the insured population.

For these reasons, we use general population data for mortality forecasting. Relational models then allow us to take adverse selection into account. Specifically, the overall mortality trend is estimated from the general population, and a regression model is then used to switch from the general population to the insurance market.

The remainder of this paper is organized as follows. As any sound procedure for projecting mortality must begin with a careful analysis of past trends, Section 2 illustrates the observed decline in mortality, on the basis of mortality statistics for the Belgian general population. The mortality experience over 1950-2015 is carefully studied by means of several demographic indicators. Previous mortality projections are also compared to the actual mortality experienced over the last years. Section 3 is then devoted to mortality projections for the entire population. In Section 4, we compare mortality statistics gathered by the insurance regulatory authorities with general population figures using Hygie, which is another tool produced by Reacfin, in order to measure adverse selection. The final Section 5 briefly concludes the study.
2 Mortality trends in the general population

2.1 Data sources

2.1.1 Statistics Belgium

Statistics Belgium is the official statistical agency for Belgium. Formerly known as NIS-INS, Directoriate General Statistics Belgium is part of the Federal Public Service Economy. It is based in Brussels. Its mission is to deliver timely, reliable and relevant figures to the Belgian government, international authorities (like the EU), academics, and the public. For more information, we refer the reader to the official website at http://www.statbel.fgov.be. A national population register serves as the centralizing database in Belgium and provides official population figures. Statistics on births and deaths are available from this register by basic demographic characteristics (e.g., age, gender, marital status).


2.1.2 Federal Planning Bureau

The Federal Planning Bureau (FPB) is a public utility institution based in Brussels. The FPB makes studies and projections on socio-economic and environmental policy issues for the Belgian government. The population plays an important role in numerous themes examined by the FPB. This is why the FPB produces regularly updated projected life tables for Belgium.

The official mortality statistics for Belgium come from FPB together with Statistics Belgium. Specifically, from 1948 to 1993, annual death probabilities were computed by FPB. From 1994, annual death probabilities are computed by Statistics Belgium and published on a yearly basis. The annual death probabilities are now available for calendar years $t = 1948, 1949, \ldots, 2015$.

Remark 2.1 (Human mortality database, HMD). This data basis was launched in May 2002 to provide detailed mortality and population data to those interested in the history of human longevity. It has been put together by the Department of Demography at the University of California, Berkeley, USA, and the Max Planck Institute for Demographic Research in Rostock, Germany. It is freely available at http://www.mortality.org and provides a highly valuable source of mortality statistics.

HMD contains original calculations of death rates and life tables for national populations, as well as the raw data used in constructing those tables. For pricing and reserving in life insurance and pensions, official data are generally preferred. This is why in this work, we restrict our analysis to data provided by Statistics Belgium and FPB and do not consider HMD mortality statistics.
2.2 Age-Period life tables

Here, we analyze mortality in an age-period framework. This means that we use two dimensions: age and calendar time. Both age and calendar time can be either discrete or continuous variables. In discrete terms, a person aged \( x \), \( x = 0, 1, 2, \ldots \), has an exact age comprised between \( x \) and \( x + 1 \). This concept is also known as “age last birthday” (that is, the age of an individual as a whole number of years, by rounding down to the age at the most recent birthday). Similarly, an event that occurs in calendar year \( t \) occurs during the time interval \([t, t + 1)\).

In line with standard actuarial notation, denote as \( q_x(t) \) the probability that an \( x \)-aged individual in calendar year \( t \) dies before reaching age \( x + 1 \). Similarly, \( p_x(t) = 1 - q_x(t) \) is the probability that an \( x \)-aged individual in calendar year \( t \) reaches age \( x + 1 \). Throughout this paper, we assume that the age-specific forces of mortality \( \mu_x(t) \) (see Delwarde and Denuit, 2005, or Pitacco et al., 2009, for a formal definition) are constant within bands of age and time, but allowed to vary from one band to the next, i.e.

\[
\mu_{x+\xi}(t+\tau) = \mu_x(t) \quad \text{for all integers } x \text{ and } t
\]

and for all \( \xi, \tau \in (0, 1) \).

2.3 Death rates

When working with death rates, the appropriate notion of risk exposure is the person-years of exposure, called the (central) exposure-to-risk in the actuarial literature. The exposure-to-risk refers to the total number of “person-years” in a population over a calendar year. It is similar to the average number of individuals in the population over a calendar year adjusted for the length of time they are in the population.

Let \( L_{xt} \) be the number of individuals aged \( x \) last birthday on January 1 of year \( t \). This quantity is sometimes referred to as the initial exposure-to-risk and remains meaningful as long as the population considered is closed, in the sense that the only way to leave it is in case of death.

\( ETR_{xt} \) be the exposure-to-risk at age \( x \) last birthday during year \( t \), i.e. the total time lived by people aged \( x \) last birthday in calendar year \( t \). This quantity is sometimes referred to as the central exposure-to-risk, to distinguish it from \( L_{xt} \). Contrarily to \( L_{xt} \), it can cope with open groups as long as the decrements are not related to mortality. For this reason, \( ETR_{xt} \) is more appropriate to insurance mortality studies, as portfolios generally constitute open groups.

Provided the population size is large enough, the approximation

\[
ETR_{xt} \approx \frac{-L_{xt}q_x(t)}{\ln(1 - q_x(t))}
\]  

(2.1)

is reasonably accurate and can be used to reconstitute the \( ETR_{xt} \)’s from the \( L_{xt} \)’s and the \( q_x(t) \)’s in the case where the \( ETR_{xt} \)’s are not readily available. This formula appears to be
useful since, in the majority of the applications to general population data, the exposure-to-risk is not provided. When the actuary works with market data, or with statistics gathered from a given insurance portfolio, the exposures-to-risk are easily calculated so that there is no need for the approximation formula (2.1).

Crude (i.e., unsmoothed) death rates are obtained by dividing the number $D_{xt}$ of deaths recorded at age $x$ in year $t$ by the corresponding exposure-to-risk $ETR_{xt}$:

$$\hat{m}_x(t) = \frac{D_{xt}}{ETR_{xt}}. \quad (2.2)$$

The death rate is thus the proportion of people of a given age expected to die within the year, expressed in terms of the expected number of life-years rather than in terms of the number of individuals initially present in the group.

Figure 2.1 displays the logarithm of the death rates $\hat{m}_x(t)$ for males and females for four selected periods covering the last 65 years: 1955, 1975, 1995, and 2015. For each period, death rates are relatively high in the first year after birth, decline rapidly to a low point around age 10, and thereafter rise, in a roughly exponential fashion, before decelerating (or slowing their rate of increase) at the end of the life span. This is the typical shape of a set of death rates.

As we can observe on Figure 2.1, the hump in mortality around ages 18-25 has become increasingly important, especially for young males. Accidents, injuries, and suicides account for the majority of the excess mortality of males over females at ages under 45 (this is why this hump is often referred to as the accident hump). We also observe a suspect hump for male at age 99 during the year 1955. This incidental hump can be explained by a number of males aged 99 on January 1st of 1955 equal to the number of recorded deaths in this population during that year. It gives an example of case where data smoothing and mortality table closure can be very interesting, as it will be discussed further.

### 2.4 Mortality surfaces

The dynamic analysis of mortality is often based on the modelling of the mortality surfaces that are depicted in Figure 2.2. We can see there the typical dialog boxes and figures produced by the Reacfin’s tool Thanatos. A mortality surface consists of a 3-dimensional plot of the logarithm of the $\hat{m}_x(t)$’s viewed as a function of both age $x$ and time $t$. Fixing the value of $t$, we recognize the classical shape of a mortality curve visible on Figure 2.1. Specifically, along cross sections when $t$ is fixed (or along diagonals when cohorts are followed), one observes relatively high mortality rates around birth, the well-known presence of a trough at about age 10, a ridge in the early 20’s (which is less pronounced for females), and an increase at middle and older ages.

We see from Figure 2.2 that data at old ages produce suspect results (because of small risk exposures): the pattern at old and very old ages is heavily affected by random fluctuations because of the scarcity of data. Sometimes, data above some high age are not available at all. Recently, the study conducted by Gbari et al. (2017) has provided a more sound knowledge about the slope of the mortality curve at very old ages. We do not address this issue here and we concentrate on the age range going from 0 to 99.
Figure 2.1: Death rates (on the log scale) for Belgian males (top panel) and Belgian females (bottom panel) from period life tables 1955, 1975, 1995, and 2015. Source: FPB.
Figure 2.2: Observed death rates (on the log scale) for Belgian males (top panel) and Belgian females (bottom panel), ages 0 to 99, period 1950-2015. Source: FPB.
2.5 Life expectancies

Period life expectancies are a useful measure of the mortality rates that have been actually experienced over a given period and, for past years, provide an objective means of comparison of the trends in mortality over time. Let \( \tau_x^t \) be the period life expectancy at age \( x \) in calendar year \( t \). Here, we have used a superscript “↑” to recall that we work along a vertical band in the Lexis diagram, considering death rates associated with a given period of time. Specifically, \( \tau_x^t \) is computed from the period life table for year \( t \), given by the set \( q_{x+k}(t) \), \( k = 0, 1, \ldots \).

Figures 2.3 and 2.4 show the trend in the period life expectancies at birth \( \tau_0^t \) and at retirement age \( \tau_{65}^t \) by gender. The period life expectancy at a particular age is based on the death rates for that and all higher ages that were experienced in that specific year. For life expectancies at birth, we observe a regular increase whereas remaining life expectancies at age 65 only started to increase after 1970.

2.6 Heterogeneity

Within populations, differences in life expectancy exist with regard to gender. Females tend to outlive males in all populations, and have lower mortality rates at all ages, starting from infancy. This is clear from all of the figures examined so far in this chapter. Another difference in life expectancy occurs because of social class, as assessed through occupation, income, or education.

In general, individuals with higher socio-economic status live longer than those in lower socio-economic groups. Blanpain and Chardon (2011) have computed mortality differentials in France, based on data provided by INSEE. Retirees are classified according to their occupation before retirement. The Standardized Mortality Ratio (SMR) is a useful index for comparing mortality experiences: actual deaths in a particular population are compared with those which would be expected if “standard” age-specific rates applied. Table 2.1 summarizes the values of SMR according to the socio-economic group in the French population and its evolution through time. In this table, out of the labour force covers individuals who never worked whereas unemployed individuals are ranked according to their last occupation. As we can observe, out of the labour individuals have the highest ratios. This observation can be explained by the fact that the individuals who never worked are mainly disabled people who have higher mortality rates. On the other hand, farming and executive management seem to be the safest socio-professional categories for young men while executive management takes the lead at older ages.
Figure 2.3: Observed period life expectancies at birth (top panel) and at age 65 (bottom panel) for Belgian males, period 1950-2015. Source: FPB.
Figure 2.4: Observed period life expectancies at birth (top panel) and at age 65 (bottom panel) for Belgian females, period 1950-2015. Source: FPB.
Table 2.1: SMR according to the socio-economic group in the French population and resulting differentials in remaining life expectancy at age 35. Source: Blanpain and Chardon (2011).

We will see in Section 4 that the effect of social class is significant for insurance market mortality statistics. Indeed, the act of purchasing life insurance products often reveals that the individual belongs to upper socio-economic class, which in turn yields lower mortality (even in the case of death benefits).

The observed heterogeneity inside populations clearly shows the need for risk classification in life insurance. This has been clearly demonstrated by Cossette et al. (2007) on a database from the Régie des Rentes du Québec (RRQ) as well as by Delwarde and Denuit (2004) on a database from the Belgian Ministry of Finance about pensions paid to Belgian civil servants. The main findings were each time the strong heterogeneity and the presence of numerous relevant risk factors. See also Delwarde et al. (2004) for a study conducted with the French
reinsurer SCOR and Gschlossl et al. (2011) with the German reinsurer Munich Re. Classical risk factors include Education level (Secondary, Post secondary, Graduate school, master), Income, Marital status, Smoking status, Drinking habit, Obesity (Underweight, Normal, Overweight), Physical activity, etc. Whereas risk classification has been mainly confined to nonlife insurance up to now, we believe that actuaries working in life insurance should pay more attention to the effect of such risk factors when they manage products that are sensitive to the life tables.

Remark 2.2. Actuaries sometimes weight their calculations by policy size to account for socio-economic differentials amongst policyholders. These “amount-based” measures usually produce lower mortality rates than their “lives-based” equivalents due to the tendency for wealthier policyholders to live longer. The pension size is thus used as a proxy of socio-economic group. However, this approach is somewhat ad-hoc, and the amount of pension should better be included explicitly as a covariate in the regression models used for mortality projections. We refer the reader to Denuit and Legrand (2016) for an appropriate method to include amounts insured in mortality modelling.

3  Mortality projections, general population

3.1 Lee-Carter mortality projection model

3.1.1 Genesis

There is now a variety of statistical models available for mortality projection, ranging from the basic regression models, in which age and time are viewed as continuous covariates, to sophisticated robust nonparametric models. In this paper, we consider the log-bilinear projection model pioneered by Lee and Carter (1992) that has now been widely adopted. However, it is of course not the only candidate for extrapolating mortality to the future. It should be stressed that some models are designed to project specific demographic indicators, and that the forecast horizon may depend on the type of model. In this respect, the Lee-Carter model is typically meant for long-term projections of aggregate mortality indicators like life expectancies. It is not intended to produce reliable forecasts of series of death rates for a particular age. This is why this model is so useful for actuaries, who are interested in life annuity premiums and reserves, which are weighted versions of life expectancies (the weights being the financial discount factors).

It should be noted that the mortality projection model used in this paper does not attempt to incorporate assumptions about advances in medical science or specific environmental changes: no information other than previous history is taken into account. The (tacit) underlying assumption is that all of the information about the future is contained in the past observed values of the death rates. This means that this approach is unable to forecast sudden improvements in mortality due to the discovery of new medical treatments, revolutionary cures including antibiotics, or public health innovations. Similarly, future deteriorations caused by epidemics, the appearance of new diseases or the aggravation of pollution cannot enter the model. The actuary has to keep this in mind when he uses this model and makes decision on the basis of the outputs, for example in the setting of a reinsurance program.
Some authors have severely criticized the purely extrapolative approach because it seems to ignore the underlying mechanisms of a social, economic or biological nature. As pointed out by Wilmoth (2000), such a critique is valid only insofar as such mechanisms are understood with sufficient precision to offer a legitimate alternative method of prediction. Since our understanding of the complex interactions of social and biological factors that determine mortality levels is still imprecise, the extrapolative approach to prediction is still compelling in the case of human mortality.

3.1.2 Specification

Lee and Carter (1992) specified a log-bilinear form

\[ \alpha_x + \beta_x \kappa_t. \]  

(3.1)

for the force of mortality on the log-scale. The specification (3.1) differs structurally from parametric models given that the dependence on age is nonparametric, and represented by the sequences of \( \alpha_x \)'s and \( \beta_x \)'s. Interpretation of the parameters is quite simple: \( \exp \alpha_x \) is the general shape of the mortality schedule and the actual forces of mortality change according to an overall mortality index \( \kappa_t \) modulated by an age response \( \beta_x \) (the shape of the \( \beta_x \) profile tells which rates decline rapidly and which slowly over time in response of change in \( \kappa_t \)). The parameter \( \beta_x \) represents the age-specific patterns of mortality change. It indicates the sensitivity of the logarithm of the force of mortality at age \( x \) to variations in the time index \( \kappa_t \).

Remark 3.1. Considering the global convergence in mortality levels, and the common trends evidenced in Pitacco et al. (2009), it may seem appropriate to prepare mortality forecasts for individual national populations in tandem with one another. For producing mortality forecasts for a group of populations, the central tendencies for the group are first identified using a common factor approach, and national historical particularities are then taken into account. See e.g. Delwarde et al. (2006) for an analysis of mortality decline in the G5 countries (France, Germany, Japan, UK and US) as well as Antonio et al. (2015).

3.1.3 Calibration

Identifiability constraints In (3.1), the \( \alpha_x \) parameters can only be identified up to an additive constant, the \( \beta_x \) parameters can only be identified up to a multiplicative constant, and the \( \kappa_t \) parameters can only be identified up to a linear transformation. Precisely, if we replace \( \beta_x \) with \( c \beta_x \) and \( \kappa_t \) with \( \frac{\kappa_t}{c} \) for any \( c \neq 0 \) or if we replace \( \alpha_x \) with \( \alpha_x - c \beta_x \) and \( \kappa_t \) with \( \kappa_t + c \) for any \( c \), we obtain the same values for the death rates. This means that we cannot distinguish between the two parametrizations: different values of the parameters produce the same mortality. A pair of additional constraints are required on the parameters for estimation to circumvent this problem. To some extent, the choice of the constraints is a subjective one, although some choices are more natural than others. In the literature, the parameters in (3.1) are usually subject to the constraints

\[ \sum_t \kappa_t = 0 \text{ and } \sum_x \beta_x = 1 \]  

(3.2)
ensuring model identification. Under this normalization, $\beta_x$ is the proportion of change in the overall log mortality attributable to age $x$.

Note that the lack of identifiability of the Lee-Carter model is not a real problem. It just means that the likelihood associated with the model has an infinite number of equivalent maxima, each of which would produce identical forecasts. Adopting the constraints \((3.2)\) consists in picking one of these equivalent maxima. The important point is that the choice of constraints has no impact on the quality of the fit, or on forecasts of mortality.

**Estimation** Regression models treating age and calendar time as factors are generally used to extract the $\alpha_x$, $\beta_x$ and $\kappa_t$ from the available mortality statistics. The products $\beta_x \kappa_t$ make them nonlinear so that standard GLM packages cannot be used. Different approaches have been proposed so far:

- **OLS, SVD**: \( \ln \hat{\mu}_x(t) = \alpha_x + \beta_x \kappa_t + \varepsilon_x(t) \) with $\varepsilon_x(t) \sim \mathcal{N}(0, \sigma^2)$
  Lee and Carter (1992)
- **Poisson**: \( D_{xt} \sim \text{Poi}(\text{ETR}_x \exp (\alpha_x + \beta_x \kappa_t)) \)
  Brouhns et al. (2002a,b)
- **Binomial**: \( D_{xt} \sim \text{Bin}(L_{xt}, 1 - \exp (-\exp (\alpha_x + \beta_x \kappa_t))) \)
  Cossette et al. (2007)
- **Neg. Bin.**: \( D_{xt} \sim \text{NB}in(\text{ETR}_x \exp (\alpha_x + \beta_x \kappa_t), \tau) \)
  Delwarde et al. (2007b).

**Remark 3.2 (Adjustment of the OLS $\hat{\kappa}_t$).** Instead of keeping the $\hat{\kappa}_t$’s obtained from singular value decomposition or Newton-Raphson algorithm, Lee and Carter (1992) suggested that the $\hat{\kappa}_t$’s (taking the $\hat{\alpha}_x$’s and $\hat{\beta}_x$’s as given) be adjusted in order to reproduce the observed number of deaths $\sum_x D_{xt}$ in year $t$. This avoids discrepancies arising from modelling on the logarithmic scale. Whereas Lee and Carter (1992) have suggested that the $\hat{\kappa}_t$ be adjusted by refitting to the total observed deaths, Lee and Miller (2001) have proposed an adjustment procedure in order to reproduce the period life expectancy at some selected age (instead of the total number of deaths recorded during the year). The advantage of this second adjustment procedure is that it does not require exposures-to-risk nor death counts and is thus generally applicable. Booth et al. (2002) have suggested another procedure for adjusting the $\hat{\kappa}_t$’s. Rather than fitting the yearly total number of deaths, this variant fits to the age distribution of deaths $D_{xt}$ assuming the Poisson distribution for the age-specific death counts and using the deviance statistic to measure the goodness-of-fit. In contrast to the classical least squares approach to estimating the parameters, the error applies directly on the number of deaths in the Poisson regression approach. There is, thus, no need for a second-stage estimation for the $\kappa_t$’s in this case as likelihood equations ensure that the estimated $\kappa_t$’s are such that the resulting death rates applied to the actual risk exposure produce the total number of deaths actually observed in the data for each age $x$. Sizable discrepancies between predicted and actual deaths are thus avoided.

**Results** We now fit the log-bilinear model to the FPB data set by Poisson maximum likelihood. The calendar years 1950-2015 and ages 0-99 are first included in the analysis.
Figure 3.1 (top panels) displays the estimated $\alpha_x$’s, $\beta_x$’s and $\kappa_t$’s for males. The estimated $\alpha_x$’s exhibit the typical shape of a set of log death rates with relatively high values around birth, a decrease at infant ages, the accident hump, and finally the increase at adult ages with an ultimately concave behaviour. The estimated $\beta_x$’s appear to globally decrease with age (with an exception of tenuous increase between 20 years and the age of retirement) suggesting that most of the mortality decreases are concentrated on the young ages. Restricting to ages above 60, Figure 3.1 (bottom panels) plots the estimated $\alpha_x$, $\beta_x$ and $\kappa_t$. The fitted mortality surfaces are also depicted in Figure 3.1. These surfaces should be compared with Figure 2.2.

3.2 Smoothing

3.2.1 Motivation

Actuaries use projected life tables in order to compute life annuity prices, life insurance premiums as well as reserves. Any irregularities in these life tables would then be passed on to the price list and to balance sheets, which is not desirable. Therefore, as long as these irregularities do not reveal particular features of the risk covered by the insurer, but are likely to be caused by sampling errors, actuaries prefer to resort to statistical techniques in order to produce life tables that exhibit a regular progression, in particular with respect to age.

Death rates can be smoothed with P-splines in the context of a Poisson model. P-splines, or penalized B-splines, use B-splines with a discrete penalty on the differences of their coefficients. The P-spline approach is an example of a regression model and is similar to the generalized linear modelling. But unlike generalized linear models, P-splines allow for more flexibility in modelling observed mortality. The P-spline approach is also useful as a graduation method based on 2-dimensional splines, to remove random noise from the mortality surface. Besides producing a smooth mortality surface, this approach has the considerable advantage of imposing no preconception about “shape” on the graduation. The results produced are entirely driven by the data. Moreover, the method does not over-smooth the data and so does not remove features in the data that may interest actuaries.

Figure 3.2 (middle panel) shows the obtained mortality surface using optimal smoothing parameters chosen by minimization of the Bayesian Information Criterion (BIC) and compares it to the one obtained via local polynomial regression (top panel). The mortality surface appears to be significantly smoother at higher ages due to the fact that the P-spline approach uses the logarithm of the exposures-to-risk matrix as offset and gives thus less weight to these data when performing smoothing.

Another smoothing method consists in a discrete non-parametric graduation of death rates using the discrete beta kernel estimator, also called the Nadaraya-Watson kernel estimator. This graduation uses as parameters global bandwidths for ages and years which can be understood as compromises between bias and variability of the distribution. In this way, higher values of the age bandwidth lead to smoother results for the mortality surface at higher ages. As with P-splines, the results here are driven by the data and use the exposures-to-risk matrix to dynamically adapt the global bandwidths at each age and at each year according to the reliability of the data.

Figure 3.2 (bottom panel) shows the resulting mortality surface where adaptive smoothing
Figure 3.1: Estimated $\alpha_x, \beta_x$ and $\kappa_t$ and fitted death rates (on the log scale) for Belgian males, $x = 0, 1, \ldots, 99$ (top panels) and $x = 60, 61, \ldots, 99$ (bottom panels), $t = 1950, 1951, \ldots, 2015$, obtained with FPB data.
parameters are determined using cross-validation. This surface is smoother than the one obtained with local polynomial regression (top panel) but less smooth than the one obtained using the P-splines approach (middle panel).

3.2.2 Smoothing in the Lee-Carter model

As can be seen from Figure 3.1, the estimated $\beta_x$’s exhibit an irregular pattern. This is undesirable from an actuarial point of view, since the resulting projected life tables will also show some erratic variations across ages. Note that the estimated $\alpha_x$’s are usually very smooth, since they represent an average effect of mortality at age $x$. The estimated $\kappa_t$’s are often rather irregular, but the projected $\kappa_t$’s, obtained from some time series model (as explained below), will be smooth. Hence, we only need to smooth the $\beta_x$’s in order to get projected life tables with mortality varying smoothly across the ages. Delwarde et al. (2007a) smoothed the $\beta_x$’s by penalized least squares or maximum log-likelihood methods where the objective function can be seen as a compromise between goodness-of-fit (first term) and smoothness of the $\beta_x$’s (second term). The penalty involves the sum of the squared second order differences of the $\beta_x$’s, that is, the sum of the squares of $\beta_{x+2} - 2\beta_{x+1} + \beta_x$. Second order differences penalize deviations from the linear trend. The trade off between fidelity to the data (governed by the sum of squared residuals) and smoothness (governed by the penalty term) is controlled by the smoothing parameters $\pi_\beta$. The larger the smoothing parameters the smoother the resulting fit. In the limit ($\pi_\beta \rightarrow \infty$) we obtain a linear fit. The choice of the smoothing parameters is crucial as we may obtain quite different fits by varying the smoothing parameters $\pi_\beta$. The choice of the optimal $\pi_\beta$ is based on the observed data, using cross-validation techniques. Penalized least squares is similar to Whittaker-Henderson graduation, a non-parametric graduation method that has been commonly used in the US.

3.2.3 Results

Figure 3.3 (top panel) plots the estimated $\alpha_x$, $\beta_x$ and $\kappa_t$ obtained by the penalized least squares method where no prior smoothing of the mortality surface has been done. The optimal smoothing parameter found using cross-validation techniques is $10^8$. Figure 3.4 (top panel) plots the estimated $\alpha_x$, $\beta_x$ and $\kappa_t$ obtained by the maximum penalized log-likelihood method. In this case, the optimal smoothing parameter found using cross-validation techniques is also $10^8$.

We see that smoothing does not impact on the estimated $\alpha_x$’s displayed on Figure 3.1, except just before the accident hump, nor on the estimated $\kappa_t$’s. Smoothing does however impact on the estimated $\beta_x$’s which now appear to behave very regularly with age.

We now restrict ourselves to ages above 60. Figure 3.3 (bottom panel) plots the estimated $\alpha_x$, $\beta_x$ and $\kappa_t$ obtained by the penalized least squares method. The optimal smoothing parameter found using cross-validation techniques is $10^8$. Figure 3.4 (bottom panel) plots the estimated $\alpha_x$, $\beta_x$ and $\kappa_t$ obtained by the maximum penalized log-likelihood method. The optimal smoothing parameter found using cross-validation techniques is $10^7$. We see that smoothing has almost no impact on the estimated $\alpha_x$’s nor on the estimated $\kappa_t$’s, whereas the estimated $\beta_x$’s are smoothed in an appropriate way.
Figure 3.2: Comparison between mortality surfaces smoothed via local polynomial regression (top panel), using P-splines (middle panel) and via discrete beta kernel estimator (bottom panel) for Belgian males.
Figure 3.3: Estimated $\alpha_x$, $\beta_x$ and $\kappa_t$ (from left to right), $x = 0, 1, \ldots, 99$ (top panel) and $x = 60, 61, \ldots, 99$ (bottom panel), $t = 1950, 1951, \ldots, 2015$, obtained by penalized least squares.
Figure 3.4: Estimated $\alpha_x$, $\beta_x$ and $\kappa_t$ (from left to right), $x = 0, 1, \ldots, 99$ (top panel) and $x = 60, 61, \ldots, 99$ (bottom panel), $t = 1950, 1951, \ldots, 2015$, obtained by maximum penalized log-likelihood.
3.3 Selection of an optimal calibration period

Many actuarial studies have based the projections of mortality on the statistics relating to the years from 1950 to the present. The question then becomes why the post 1950 period better represents expectations for the future than does the post 1900 period, for example. There are several justifications for the use of the second half of the 20th century. First, the pace of mortality decline was more even across all ages over the 1950-2000 period than over the 1900-2000 period. Secondly, the quality of mortality data, particularly at the older ages, for the 1900-1950 period is questionable. Thirdly, infectious diseases were an uncommon cause of death by 1950, while heart disease and cancer were the two most common causes, as they are today. This view seems to imply that the diseases affecting death rates from 1900 through 1950 are less applicable to expectations for the future than the dominant causes of death from 1950 through 2000.

According to Lee and Carter (1992), the length of the mortality time series was not critical as long as it was more than about 10-20 years. However, Lee and Miller (2001) obtained better fits by restricting the start of the calibration period to 1950 in order to reduce structural shifts. Specifically, in their evaluation of the Lee-Carter method, Lee and Miller (2001) have noted that for US data the forecast was biased when using the fitting period 1900-1989 to forecast the period 1990-1997. The main source of error was the mismatch between fitted rates for the last year of the fitting period (1989 in their study) and actual rates in that year. This is why a bias correction is applied. It was also noted that the $\beta_x$ pattern did not remain stable over the whole 20th century. In order to obtain more stable $\beta_x$’s, Lee and Miller (2001) have adopted 1950 as the first year of the fitting period. Their conclusion is that restricting the fitting period to 1950 on avoids outlier data.

Booth et al. (2002) have designed procedures for the selection of an optimal calibration period which identifies the longest period for which the estimated mortality index parameter $\kappa_t$ is linear. Specifically, Booth et al. (2002) assume, a priori, that the trend in the adjusted $\hat{\kappa}_t$’s is linear. When the $\hat{\kappa}_t$’s depart from linearity, this assumption may be better met by appropriately restricting the fitting period. The choice of the fitting period is based on the ratio of the mean deviances of the fit of the underlying Lee-Carter model to the overall linear fit. This ratio is computed by varying the starting year (but holding the jump-off year fixed) and the chosen fitting period is that for which the ratio is substantially smaller (or closer to one in our specific case of smaller-than-one ratios) than for periods starting in previous years.

The ending year is kept equal to 2015 and the fitting period is then determined by the starting year (henceforth denoted as $t_{start}$). Restricting the fitting period to the longest recent period ($t_{start}, 2015$) for which the adjusted $\hat{\kappa}_t$’s do not deviate markedly from linearity has several advantages. Since systematic changes in the trend in $\hat{\kappa}_t$ are avoided, the uncertainty in the forecast is reduced accordingly. Moreover, the $\beta_x$’s are likely to satisfy better the assumption of time invariance. Finally, the estimates of the projection parameters more clearly reflect the recent experience.

An ad-hoc procedure for selecting $t_{start}$ has been suggested in Denuit and Goderniaux (2005). Precisely, the calendar year $t_{start} \geq 1950$ is selected in such a way that the series $\{\hat{\kappa}_t, t = t_{start}, t_{start}+1, \ldots , 2015\}$ is best approximated by a straight line. To this end, the adjustment coefficient $R^2$ (which is the classical goodness-of-fit criterion in linear regression)
is maximized (as a function of the number of observations included in the fit).

### 3.3.1 Application to Belgian mortality statistics

We first consider the Lee-Carter fit. Applying the method of Booth et al. (2002) for Belgian males gives $t_{\text{start}} = 1993$. However, the ad-hoc method suggested in Denuit and Goderniaux (2005) suggests that we consider a period starting at $t_{\text{start}} = 1980$. In order to choose between those two possibilities, we analyse the graph of the mean deviances computed using Booth et al. (2002) method and displayed on Figure 3.5 (top left panel). We observe that the ratio of deviances remains close to 99% after 1980. We will therefore choose as optimal starting year the year which maximises the adjustment coefficient $R^2$, so $t_{\text{start}} = 1980$.

Restricting the age range to 60 and over for Belgian males yields $t_{\text{start}} = 1967$ if Booth et al. (2002) method is applied, whereas the ad-hoc method gives also $t_{\text{start}} = 1980$. Once again, we analyse Figure 3.5 (bottom left panel) in order to choose an optimal starting year for the fitting period. Here, we clearly observe a smaller gap between the mean deviances after 1967. We therefore choose $t_{\text{start}} = 1967$ as optimal starting year in this case.

Whereas the common practice would consist of taking all of the available data 1950-2015, we discard here observations for the years 1950-1979 when all of the ages are considered, and observations for the years 1950-1966 when the analysis is restricted to ages 60 and over. Here, short-term trends are preferred even if long-term forecasts are needed for annuity pricing. The reason is that past long-term trends are not expected to be relevant to the long-term future.

The final estimates based on observations comprised in the optimal fitting period are displayed in Figure 3.6, Figure 3.7 and Figure 3.8 which plot respectively the estimated $\alpha_x$'s, $\beta_x$'s and $\kappa_t$'s obtained via the Lee-Carter model without smoothing of the estimated parameters and the Lee-Carter model where the $\beta_x$'s are smoothed via the penalized least squares method and the maximum penalized log-likelihood method. The optimal smoothing parameters are found using cross-validation techniques. We see that the estimated $\alpha_x$'s and $\kappa_t$'s obtained with and without smoothing closely agree whereas the estimated $\beta_x$'s are smoothed in an appropriate way.

### 3.4 Mortality projection

An important aspect of the Lee-Carter methodology is that the time factor $\kappa_t$ is intrinsically viewed as a stochastic process. Specifically, the estimated $\kappa_t$'s are viewed as a realization of a time series that is modelled using the classical autoregressive integrated moving average (ARIMA) models. Such models explain the dynamics of a time series by its history and by contemporaneous and past shocks. The dynamics of the $\kappa_t$'s is described by an ARIMA($p, d, q$) process if it is stationary and

$$\nabla^d \kappa_t = \phi_1 \nabla^d \kappa_{t-1} + \ldots + \phi_p \nabla^d \kappa_{t-p} + \xi_t + \psi_1 \xi_{t-1} + \ldots + \psi_q \xi_{t-q}$$

(3.3)

where $\nabla^d$ is the difference operator of degree $d$, with $\phi_p \neq 0$, $\psi_q \neq 0$, and where $\xi_t$ is a Gaussian white noise process such that $\sigma^2 > 0$.

There are a few basic steps in order to fit ARIMA models to time series data. The main point is to identify the values of the autoregressive order $p$, the order of differencing $d$, and
Figure 3.5: Mean deviances (left panels) and their ratios (right panels) for Belgian males for the age ranges $x = 0, 1, \ldots, 99$ (top panel) and $x = 60, 61, \ldots, 99$ (bottom panel), $t_{\text{start}} = 1950, 1951, \ldots, 1995$. 
Figure 3.6: Estimated $\alpha_x$, $\beta_x$ and $\kappa_t$ (from left to right), $x = 0, 1, \ldots, 99$ (top panels) and $x = 60, 61, \ldots, 99$ (bottom panels), obtained over the optimal fitting period 1980-2015 for ages 0-99 and 1967-2015 for ages 60-99, without smoothing of the parameters.
Figure 3.7: Estimated $\alpha_x$, $\beta_x$ and $\kappa_t$ (from left to right), $x = 0, 1, \ldots, 99$ (top panels) and $x = 60, 61, \ldots, 99$ (bottom panels), obtained by penalized least squares over the optimal fitting period 1980-2015 for ages 0-99 and 1967-2015 for ages 60-99.
Figure 3.8: Estimated $\alpha_x$, $\beta_x$ and $\kappa_t$ (from left to right), $x = 0, 1, \ldots, 99$ (top panels) and $x = 60, 61, \ldots, 99$ (bottom panels), obtained by maximum penalized log-likelihood over the optimal fitting period 1980-2015 for ages 0-99 and 1967-2015 for ages 60-99.
the moving average order $q$. If the time series of $\kappa_t$'s is not stationary, then a first difference (i.e. $d = 1$) can help to remove the time trend. If this proves unsuccessful then it is standard to take further differences (i.e. investigate $d = 2$ and so on). A first-order difference ($d = 1$) appears to be enough to make the $\kappa_t$'s stationary.

Figure 3.9 and 3.10 below display the projections of the time factors $\kappa_t$ previously obtained in Figure 3.7 and Figure 3.8 from optimal starting years corresponding to the years for which the adjustment coefficient $R^2$ of the linear regression on the $\kappa_t$'s going from that year to 2015 is maximal. The optimal parameters $p$ and $q$ used for this forecast were chosen by minimization of the Akaike Information Criterion (AIC). As shown on Figure 3.9 and 3.10, those parameters differ for ages between 0 and 99, depending on the method used to smooth the parameters during calibration. For the penalized least squares method over the optimal fitting period, an ARIMA(0,1,0) was chosen, whereas for the maximum penalized log-likelihood, an ARIMA(2,1,0) was used. The tables appearing in Figure 3.9 and Figure 3.10 summarize the goodness-of-fit for different values of $p$ and $q$, depending on the chosen calibration method.

3.5 Prediction intervals

In forecasting, it is important to provide information on the error affecting the forecasted quantities. In the traditional demographic approach to mortality forecasting, a range of uncertainty is indicated by high and low scenarios, around a medium forecast that is intended to be a best estimate. However, it is not clear how to interpret this high-low range unless a corresponding probability distribution is specified.

In this respect, prediction intervals are particularly useful. In the current application, it is impossible to derive the relevant prediction intervals analytically. The reason for this is that two very different sources of uncertainty have to be combined: sampling errors in the parameters $\alpha_x$, $\beta_x$, and $\kappa_t$, and forecast errors in the projected $\kappa_t$'s. An additional complication is that the measures of interest – life expectancies or life annuities premiums and reserves – are complicated non-linear functions of the parameters $\alpha_x$, $\beta_x$, and $\kappa_t$ and of the ARIMA parameters. The key idea behind the bootstrap is to resample from the original data (either directly or via a fitted model) in order to create replicate data sets, from which the variability of the quantities of interest can be assessed. Because this approach involves repeating the original data analysis procedure with many replicate sets of data, it is sometimes called a computer-intensive method. Bootstrap techniques are particularly useful when, as in our problem, theoretical calculation with the fitted model is too complex.

If we ignore the other sources of errors, then the confidence bounds on future $\kappa_t$'s that are visible in Figures 3.9 and 3.10 can be used to calculate prediction intervals for demographic indicators. Even if for long-run forecasts (over 25 years), the error in forecasting the mortality index clearly dominates the errors in fitting the mortality matrix, prediction intervals based on $\kappa_t$ alone may understate the errors in forecasting over shorter horizons. We know from Lee and Carter (1992, Appendix B), that prediction intervals based on $\kappa_t$ alone are a reasonable approximation only for forecast horizons greater than 10 to 25 years. If there is a particular interest in forecasting over the shorter term, then we cannot make a precise analysis of the forecast errors. We refer the reader to Brouhns et al. (2002a, b, 2005) and to Koissi et al. (2006) for more details.
Figure 3.9: Projection of the time parameters $\kappa_i$'s, for ages $x = 0, 1, \ldots, 99$ (top panel) and $x = 60, 61, \ldots, 99$ (bottom panel), obtained by penalized least squares.
Figure 3.10: Projection of the time parameters $\kappa_i$’s, for ages $x = 0, 1, \ldots, 99$ (top panel) and $x = 60, 61, \ldots, 99$ (bottom panel), obtained by maximum penalized log-likelihood.
3.6 Closing the mortality surface

As discussed in Remark 3.1, high ages where few people are left alive generate thus very volatile observations. It is therefore necessary to use a closure model in order to have reliable mortality features and to prevent the kind of suspect results we had previously observed on Figure 2.2. To achieve this, four models are available in Thanatos.

The first closing method, which is also the easiest to compute, consists in extending our last observed death rates from the age they were observed to the desired ultimate age. This method is generally safe in an insurance framework as it underestimates death probabilities and as the majority of policies at high ages are annuities to pay until the time of death. Exceptions would be products as whole life insurance contracts. For these products or in order to have a more accurate model, the insurer should use one of the closure models described below. Figure 3.11 illustrates the impact of this closure extended until age 130 on the mortality surface obtained by maximum penalized log-likelihood from 1980 to 2015.

Another method introduced by Denuit and Goderniaux (2005) consists in considering that the logarithm of the death probabilities follows a quadratic function of the age while imposing the death of the entire population at a certain ultimate age and an inflexion constraint at that age. Figure 3.12 displays the obtained mortality surface when this method is applied on the age range going from 90 to 130 to a mortality surface obtained by maximum penalized log-likelihood from 1980 to 2015.

With the third method, the ultimate age is not necessarily the same for all years anymore. It is chosen by minimizing the sum of squares of the estimation errors when the logarithm of the death probabilities are considered to follow a certain function of this ultimate age. Figure 3.13 illustrates the result of this closure on a mortality surface obtained by maximum penalized log-likelihood from 1980 to 2015.

The last method provided by Thanatos is based on the Kannisto model which is calibrated on an age range chosen by the user. There is no fixed ultimate age for this method. Figure 3.14 illustrates this closure calibrated on ages going from 75 to 90 and extended until age 130 on the mortality surface obtained by maximum penalized log-likelihood from 1980 to 2015.

3.7 Back testing

In this section, we consider the computation of projected life expectancies at retirement age 65, obtained from the Lee-Carter approach, by replacing the death rates with their forecasted values. Moreover, the results are then compared with other projections performed for the Belgian population.

Let us now forecast the period life expectancies for calendar years 1981-2015, 1991-2015 and 2001-2015 on the basis of the observations relating to calendar years 1950-1980, 1950-1990 and 1950-2000, respectively. We thus investigate the predictive power of the Lee-Carter approach if it were applied in the past to forecast future mortality.

Figure 3.15 displays the forecast of the period life expectancies at age 65, together with observed values and 90% prediction intervals (grey areas, with progressively heavier shading). Considering males, we see that using 1950-1980 data gives a point forecast far below the actual life expectancies observed during 1981-2005. The Lee-Carter model would thus clearly
Figure 3.11: Closure of the mortality surface, using constant rates for ages $x = 99, 100, \ldots, 130$.

Figure 3.12: Closure of the mortality surface, using Denuit-Goderniaux model from age 90 up to age 130.
Figure 3.13: Closure of the mortality surface, using the free ultimate age approach.

Figure 3.14: Closure of the mortality surface, using Kannisto model up to age 130 and calibrated on $x = 75, 76 \ldots, 90$.  

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have underestimated the actual gains in longevity after 1980 on the basis of the 1950-1980 observation period. The forecast becomes better when the 1950-1990 and 1950-2000 periods are considered. For females, the mortality projections appear to be more reliable. The Lee-Carter model captures the trends in the observed period life expectancies, which remain in the prediction intervals.

4 Life insurance market

4.1 Source of data

In addition to general population data, we also analyze mortality statistics from the Belgian insurance market. Any difference between the general population and the insured population is due to adverse selection.

Market data are provided by the National Bank of Belgium (NBB) and published on http://www.nbb.be. Aggregated statistics about numbers of deaths and exposures are available there over the last five years. Annual tabulations of the number of deaths by age, by gender, and by policy type are made by the NBB based on information supplied by insurance companies. Together with the number of deaths, the corresponding risk exposure is also available in each case. These data allow us to calculate age-gender-type-of-product specific (central) death rates. We do not question the quality of the data provided by NBB. In particular, we do not explicitly deal with multi-detention, a phenomenon known to increase standard errors without biasing point estimates (see, e.g., Denuit, 2000).

When studying mortality trends on the insurance market, the actuary has always to keep in mind that possible changes in underwriting practices or tax reforms are likely to affect market death rates.

4.2 Observed death rates

Throughout this section, we will use Hygie, which is another tool produced by Reacfin, for our computations. Figure 4.1 and Figure 4.2 display the period life tables for the Belgian individual life insurance market and group life insurance market observed in the calendar years 1995, 2005, and 2015, as well as period life tables for the general population given by Thanatos and obtained by maximum penalized log-likelihood. The variability in the set of death rates is clearly much higher for the insurance market than what we had observed for the general population on Figure 2.1, as exposures-to-risk are considerably smaller here. This is why smoothing the market experience to make the underlying trend more apparent is desirable.

4.3 Life expectancies

Figure 4.3 displays the period life expectancy at age 65 for the general population and for insured lives, computed on the basis of observed death rates.

We see that the life expectancies for the group life insurance market are closer to the general population ones. This is due to the moderate adverse selection present in the collective
Figure 3.15: Observed period life expectancies at age 65 for Belgian males (top panel, black line) and Belgian females (bottom panel, black line), together with forecast based on 1950-1980 (red), 1950-1990 (green) and 1950-2000 (blue) periods surrounded by prediction 90% intervals.
Figure 4.1: General population (red lines) and individual (left panels) and group (right panels) life insurance market death rates (on the log scale) observed in 1995, 2005, and 2015 for Belgian males (blue dotted lines). Source: FPB fitted with Thanatos for the general population and NBB for insured lives.
Figure 4.2: General population (red lines) and individual (left panels) and group (right panels) life insurance market death rates (on the log scale) observed in 1995, 2005, and 2015 for Belgian females (blue dotted lines). Source: FPB fitted with Thanatos for the general population and NBB for insured lives.
contracts, where the insurance coverage is made compulsory by the employment contract, noting that there is a selection effect through being employed (the so-called “healthy worker effect”). On the contrary, the effect of adverse selection seems to be much stronger for individual policies. This is due to the particular situation prevailing in Belgium, where no tax incentives are offered for buying life annuities or other life insurance products after retirement. This explains why only people with improved health status consider insurance products as valuable assets. Note that this situation has recently changed in Belgium, where purchasing life annuities at retirement age is now encouraged by the government.

4.4 Relational models

Actuaries are aware that the nominee of a life annuity is, with a high probability, a healthy person with a particularly low mortality in the first years of life annuity payment and, generally, with an expected lifetime higher than average. In order to account for this phenomenon, Delwarde et al. (2004) have suggested a method for adjusting a reference life table to the experience of a given portfolio, based on nonlinear regression models using local likelihood for inference.

In Hygie, we assume that the number of deaths in the insured population are Poisson distributed such that

\[ D_{x,t} \sim \text{Poi} \left( ETR_{x,t} \exp \left( \beta_0 + \beta_1 \ln(\mu_{x,t}^{\text{gen pop}}) + \beta_2 x_2 \right) \right) \]

where \( x_2 \) is a characteristic function whose value depends on the considered type of life insurance (individual or group). The different parameters for the three years previously studied are firstly computed using a generalized linear model fitted on a five-year horizon around the year we consider. Figure 4.4 describes the result of the procedure for males, whereas Figure 4.5 is the analogue for females. This produces estimated SMR’s that give an indication on how mortality varies for some classes compared to the general population.

In order to assess the impact of the chosen time horizon for our specific data, we display on Figures 4.6 and 4.7 the result of the relational models whose parameters were fitted on a single year period. This fitting seems to better adjust the reference life table to the insurance market. However, the main goal of an adverse selection model is not to fit the observed data but to predict in an accurate way the future mortality of a given population.

Hence, the predictive power of such a method has to be assessed. We can observe on Hygie’s back tests for 2015 displayed on Figure 4.8 and Figure 4.9, for which the parameters of the relational models were calibrated on three different time periods (1-year, 5-year and 10-year horizon), the single year fitting seems to be the most appropriate for our dataset. Of course, this conclusion could be different for another dataset.

Figures 4.4, 4.5, 4.6, 4.7 suggest that a linear relationship exists between population and market death rates (at least for older ages), on the log-scale. This is known since Brouhns et al. (2002).

The estimated SMR’s computed on the whole period going from 1993 to 2015 are displayed in Table 4.1. The estimated SMR’s computed for each of these years are displayed in Figure 4.10.
Figure 4.3: Life expectancy at age 65 for males (top panel) and females (bottom panel): General population (black) and individual (red) and group (green) life insurance market. Source: FPB for the general population and NBB for insured lives.

<table>
<thead>
<tr>
<th></th>
<th>Individual Policies</th>
<th>Group Policies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Males</td>
<td>0.500845</td>
<td>0.6064236</td>
</tr>
<tr>
<td>Females</td>
<td>0.6363186</td>
<td>0.7138219</td>
</tr>
</tbody>
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Table 4.1: SMR’s for males and females on the whole period
Figure 4.4: Relational models for males: the resulting fits calibrated over five years around 1995, 2005 and 2015 for Belgian males are displayed in the left panels for individual policies and in the right panels for group policies.
Figure 4.5: Relational models for females: the resulting fits calibrated over five years around 1995, 2005 and 2015 for Belgian females are displayed in the left panels for individual policies and in the right panels for group policies.
Figure 4.6: Relational models for males: the resulting fits calibrated over one year (1995, 2005 and 2015) for Belgian males are displayed in the left panels for individual policies and in the right panels for group policies.
Figure 4.7: Relational models for females: the resulting fits calibrated over one year (1995, 2005 and 2015) for Belgian females are displayed in the left panels for individual policies and in the right panels for group policies.
Figure 4.8: Comparison between the backtesting of the number of deaths in 2015 with the relational model for males calibrated respectively on the period going from 2005 to 2014, from 2010 to 2014 and on the single year 2014, for individual policies (top three panels) and group policies (bottom three panels).
Figure 4.9: Comparison between the backtesting of the number of deaths in 2015 with the relational model for females calibrated respectively on the period going from 2005 to 2014, from 2010 to 2014 and on the single year 2014, for individual policies (top three panels) and group policies (bottom three panels).
Figure 4.10: Estimated SMR’s for males (top panel) and females (bottom panel), individual (red) and group (green) life insurance market.
5 Discussion and conclusions

As clearly demonstrated in this paper, mortality at adult and old ages reveals decreasing annual death probabilities throughout 1950-2015. There is an ongoing debate among demographers about whether human longevity will continue to improve in the future as it has done in the past. Some demographers argue that there is no natural upper limit to the length of human life. The approach that these demographers use is based on an extrapolation of recent mortality trends. The complexity and historical stability of the changes in mortality suggest that the most reliable method of predicting the future is merely to extrapolate past trends. However, this approach has come in for criticisms because it ignores factors relating to life style and the environment that might influence future mortality trends. Some demographers have suggested that the future life expectancy might level off or even decline. This debate clearly indicates that there is considerable uncertainty about future trends in longevity.

Longevity risk is a growing concern for insurance companies faced with off-balance-sheet or on-balance-sheet pension liabilities. More generally, all the components of social security systems are affected by mortality trends and their impact on social welfare, health care and societal planning has become a more pressing issue. And the threat has now become a reality, as testified by the failure of Equitable Life, the world’s oldest life insurance company, in the UK in 2001. Equitable Life sold deferred life annuities with guaranteed mortality rates, but failed to predict the improvements in mortality between the date the life annuities were sold and the date they came into effect.

Actuaries working in life insurance and pension have been using projected life tables for some decades. But the problem confronting actuaries is that people have been living much longer than they were expected to according to the life tables being used for actuarial computations. What was missing was an accurate estimation of the speed of the mortality improvement: thus, most of the mortality projections performed during the second half of the 20th century have underestimated the gains in longevity. The mortality improvements seen in practice have quite consistently exceeded the projected improvements. As a result, insurers have, from time to time, been forced to allocate more capital to support their in-force annuity business, with adverse effects on free reserves and profitability. From the point of view of the actuarial approach to risk management, the major problem is that mortality improvement is not a diversifiable risk. Traditional diversifiable mortality risk is the random variation around a fixed, known life table. Mortality improvement risk, though, affects the whole portfolio and can thus not be managed using the law of large numbers. In this respect, longevity resembles investment risk, in that it is non-diversifiable: it cannot be controlled by the usual insurance mechanism of selling large numbers of policies, because they are not independent in respect of that source of uncertainty. However, longevity is different from investment risk in that there are currently no large traded markets in longevity risk so that it cannot easily be hedged. The reaction to this problem is twofold. First, actuaries are trying to produce better models for mortality improvement, paying more attention to the levels of uncertainty involved in the forecasts. The second part of the reaction is to look to the capital markets to share the risk, through the emergence of mortality-linked derivatives or longevity bonds.

Despite the rapid development of mortality projection models over the last two decades, back-testing analyses have shown that most of these models failed to reproduce the evolution
of mortality over the second half of the 20th century. Actual gains in longevity would generally have been underestimated to a large extent. The break in the series of period life expectancies at retirement age that is clearly visible in the seventies in most industrialized countries is largely responsible for this failure. Moreover, projections sometimes diverge, creating a substantial model risk.

Of course, this does not mean that mortality forecasting is a vain exercise. On the contrary, it would even be worse not to project mortality at all, keeping the current survival probabilities which will be (almost surely) exceeded in the future. The failure to predict future longevity demonstrates that actuaries cannot just trust their sophisticated stochastic models blindly but that there is a need for appropriate longevity risk management tools. Projected life tables only provide actuaries with a kind of “central scenario”, from which the future mortality may (and certainly will) depart. In that respect, projected life tables do not solve the problem posed by the systematic longevity risk threatening insurers’ solvency.

Given changes in longevity and economic conditions, it appears extremely risky to specify insurance benefits in absolute terms because of the substantial financial and biometric systematic risks impacting the portfolio. Therefore, the amount of economic capital needed becomes considerable under realistic actuarial and financial models, making the products expensive and maybe not affordable at all. Linking premium and/or benefits to some appropriate indices, or modulating policy conditions (like deferment periods), is a particularly efficient strategy to counteract systematic biometric risk. Such indexing mechanisms are currently under study at KU Leuven, UCL and ULB.

We do hope that the present work will help the profession to better assess and manage the systematic biometric risk.

References


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About Reacfin

Reacfin is a consulting firm focused on setting up best-quality tailor-made Risk Management Frameworks, and offering state-of-the-art actuarial and financial techniques, methodologies & risk strategies.

While we initially dedicated ourselves to the financial services industry, we now also serve corporate or public-finance clients.

Advancements in finance and actuarial techniques are developing at a fast pace nowadays. Reacfin proposes highly-skilled and experienced practitioners, employing innovative techniques and offering expertise in compliance and risk strategies & governance. Our support will allow your firm to reach top performances and gain new competitive advantages.

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