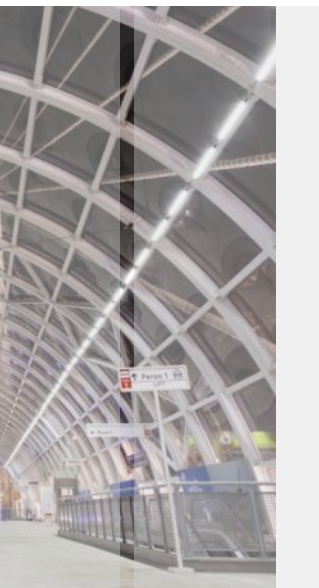


Prospective Life Tables

An introduction to time-dependent mortality models

by Julien Antunes Mendes and Christophe Pochet



TRENDS OF MORTALITY

Life expectancy at birth among early humans was likely to be about 20 to 30 years. It rose to between 40 and 45 by the middle of the 19th century. Rapid improvements began at the end of the 19th century, so that by the middle of the 20th century it was approximately 60 to 65 years. At the beginning of the 21st century, life expectancy at birth reached about 70 years. Two trends dominated the mortality decline:

- **The first half of the 20th century** saw significant improvements in the mortality of infants and children (and their mothers).
- **Since the middle of the 20th century**, gains in life expectancy have been mainly due to medical factors that have reduced mortality among older persons (reductions in deaths due to the “big three” killers – cardiovascular disease, cancer and strokes).

So, the observation demonstrates that, on average, mortality evolves with time and the future mortality rates will not be those observed today. Insurance companies should take this phenomenon into account for risk management purposes when the risk is likely to be material.

Mortality Risk

“The mortality risk is the risk of loss, or of adverse change in the value of insurance liabilities, resulting from changes in the level, trend, or volatility of mortality rates, where an increase in the mortality rate leads to an increase in the value of insurance liabilities.”

- This is a risk for the insurer if insured people die earlier than expected (typical risk for products with death benefits in which case the capital has to be paid earlier than expected by the insurer)



Indeed, when selling some life insurance contracts (eg term life insurance), the insurer is exposed to the risk that the mortality assumptions taken for the product pricing will not be in line with the mortality that will be observed in the future and could thus face issues regarding profitability.

In those products, mortality risk is borne by insurance undertakings and should thus be accounted for in the risk assessment and the calculation of the capital requirement. Solvency II explicitly recognises this risk. In the standard formula, it is even split into two components in Life and Health SLT modules: mortality and longevity risk.

Longevity Risk

“The longevity risk is associated with the risk of loss, or of adverse change in the value of insurance liabilities, resulting from changes in the level, trend, or volatility of mortality rates, where a decrease in the mortality rate leads to an increase in the value of insurance liabilities.”

- This is a risk for the insurer if insured people live longer than expected (typical risk for annuities where the insurer has to pay longer than expected)

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PERIOD AND PROSPECTIVE LIFE TABLES

There exist two main kinds of life tables:

- The period life tables are tables with mortality rates, indicating the "probability" of dying during a one year period of time, depending on the age only. So, these tables take into account that mortality is on average greater when the person is older but they do not consider that mortality evolves with time. In this case, the future mortality is supposed to be exactly the same as the mortality today.
- The prospective life tables, on the contrary, contain mortality rates depending on the age but also on the considered year.

As mortality is evolving with time, expected future changes in the mortality rates can be captured with time-depending models. The rest of this document focusses on the prospective tables models.

The models are composed of 3 kinds of parameters

- $\beta_x^{(i)}$ representing the age effect
- $\kappa_t^{(i)}$ representing the period effect
- $\gamma_{t-x}^{(i)}$ representing the cohort effect

For Lee-Carter models all parameters have to be calibrated while for CBD models age coefficients have a fixed parameterization. There are more degrees of freedom in the basic Lee-Carter model than in the basic CBD model.

Please note that these models are usually developed on general population data because specific insurer portfolios data are often too limited to produce a reliable result. Under particular assumptions and limitations, specific models can be used in order to adjust the general table into a specific insurer one, see section "Adverse selection models in insurance undertakings".

PROSPECTIVE MODELS

The main prospective life table models are part of two "families" of models: the Lee Carter and the Cairns Blake Dowd families. There are seven stochastic mortality models (3 of the Lee-Carter family and 4 of the CBD family) which involve time series analysis with parameters estimated from historical mortality rates. The coefficients of these models have to be calibrated on historical data and projection is then needed to estimate the future mortality.

- LC (Lee Carter) family, where the instantaneous mortality rate is modeled as

$$\ln m(t, x) = \sum_i \beta_x^{(i)} \kappa_t^{(i)} \gamma_{t-x}^{(i)}$$

- CBD (Cairns Blake Dowd)-Perks family, where the initial mortality rate is modeled as

$$\text{logit } q(t, x) = \sum_i \beta_x^{(i)} \kappa_t^{(i)} \gamma_{t-x}^{(i)}$$

WHICH MODEL SUITS THE BEST FOR A FIXED POPULATION

There is no universal model. The model which suits the best for a population is a model chosen according to a series of objective criteria that could depend on the situation. Analysis and tests should be carefully performed.

First of all some treatments should be performed on the historical data used to calibrate the models. These historical data should be restricted to a limited number of observed ages and times in order to improve the predictive ability of the model. If old and very old ages are under consideration, these data should be replaced by extrapolated data (involving closure models) since huge randomness is observed at these ages due to the small number of observations. The data can also be smoothed out in order to remove the undesired random variations.

Secondly, all the models can be calibrated and projected in order to determine which one will give the best fitting results. These can be compared through indicators such as Log-Likelihood, BIC, residuals...

The quality of the fit is not the only point of view to take into account as the predictive power of the model is decisive for a prospective model.

Some stress testing can allow the analysis of the robustness and stability of the coefficients' estimators of the models.

As an example, back-testing on life expectancies is also an important indicator.

Expert judgment is needed as all indicators rarely favor the same model and a trade-off has to be found.

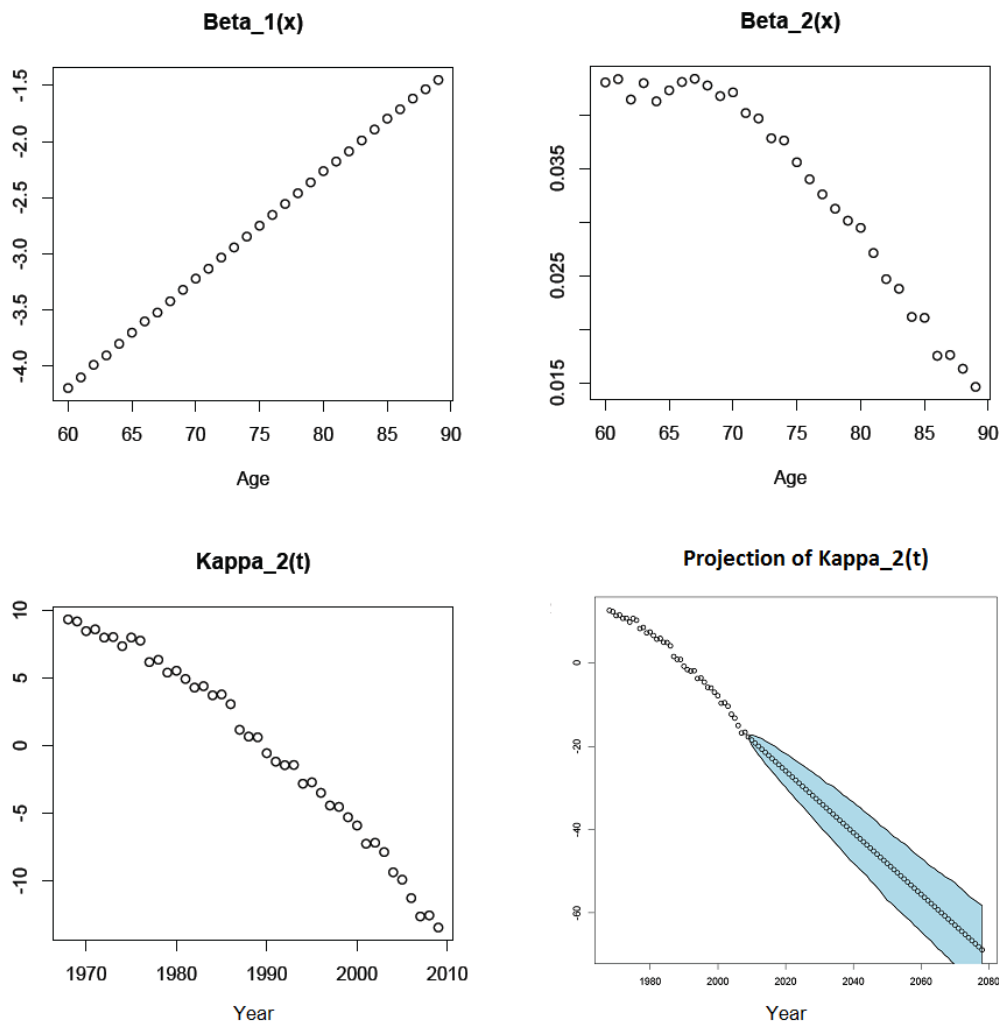


Figure 1: Calibration of the Lee-Carter parameters and projection of $\kappa_t^{(2)}$



Case study
Annuities for Belgians aged 60 or more

From a local study on the general Belgian population dedicated to compute life annuities for people aged 60 or more, it appeared that the basic Lee-Carter model was the most reliable model. The basic Lee-Carter model is surely the most adopted model on the Belgian market. It is given by:

$$\ln m(t, x) = \beta_x^{(1)} + \beta_x^{(2)} \kappa_t^{(2)}$$

This model, even if it seemed to underestimate the future life expectancy, was considered in the study as the most robust, the less volatile and the most reliable of the main models.

An illustration of the fit and projection of the Lee-Carter model parameters on the Belgian males' population between 1968 and 2009 for ages between 60 and 90 is given in Figure 1.

In order to get a projection of the mortality rates over the future years the trend component $\kappa_t^{(2)}$ should be projected. Concerning Lee-Carter Belgian model a usual market practice is to use an ARIMA(0,1,0) for this projection. Figure 1 depicts the projected values as well as a 99.5% confidence interval.

In Figure 2, the given mortality surface for the past years is represented jointly with the mortality surface projected with the central best estimate projection of the trend component $\kappa_t^{(2)}$, the parameters depending on the age remaining unchanged. The respective period life expectancy at 60-years old has been computed. Note that the full mortality surface until the maximal age is obtained thanks to a quadratic closure model.

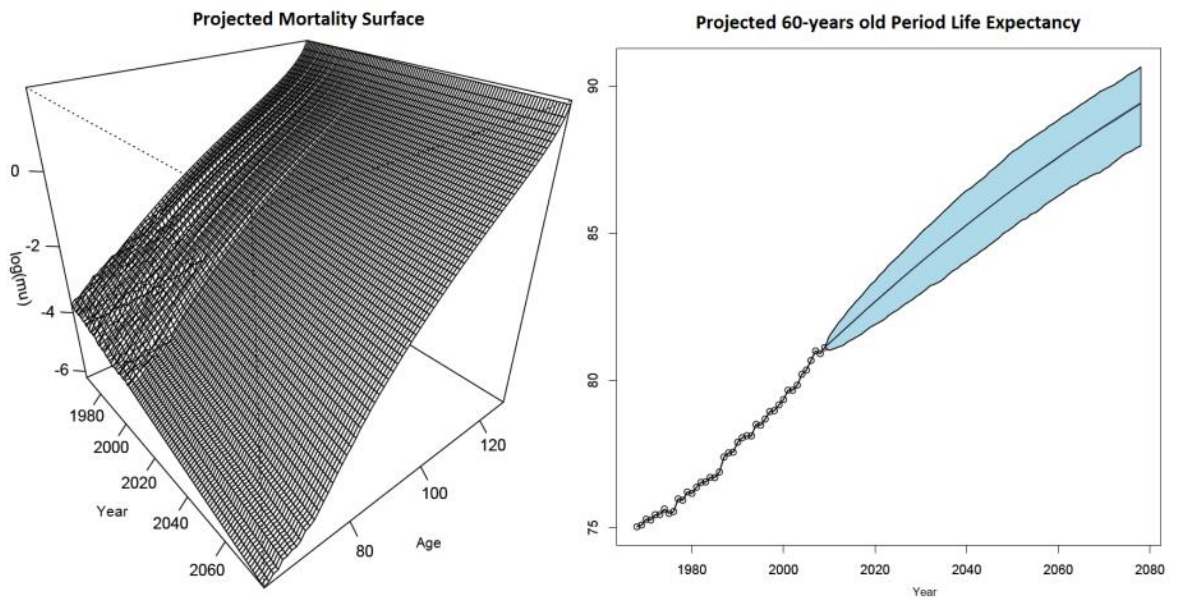


Figure 2: Projection of the mortality surface and 60-years old period life expectancy

ADVERSE SELECTION MODELS IN INSURANCE UNDERTAKINGS

Fitting prospective life table models directly on insurers' portfolios data could lead to unreliable and unstable results due to paucity of data. A typical way to tackle this issue is to fit these models on general population data and to adjust the resulting prospective life table in order to align it with insurers' portfolios' specific mortality. Note that this method is based on strong assumptions that have to be kept in mind and checked – when possible – before fitting.



The life table models coefficients' estimation is of greater quality when the input mortality data are reliable with low volatility. This is why it is advised to adopt reliable input mortality data based on a large sample of population. Estimating the coefficients directly based on the mortality data of an insurance portfolio could lead to results that are not reliable or significantly more volatile, since the number of records is often limited.

A common practice is to estimate the parameters of the models based on the general population of a given country, and as such rendering a relevant and reliable estimation.

As insurance undertakings are facing adverse selection, the mortality of the people having underwritten a specific product is not always the same as the general population mortality. For instance, it has been observed that, on average, people having underwritten annuities live longer than the general population taken as a whole. So, for risk management purposes, this feature has to be accounted for.

A common practice is to fit the prospective life table based on general population and then to adapt it in order to fit in a reliable way the mortality of the insurance portfolio. Many techniques to take adverse selection into account are developed, as relational models or adaptation models based on the life expectancies.

$$q(t, x)^{portfolio} = f(q(t, x)^{ref})$$

These models must be considered with care as they are based on strong assumptions. Due to paucity of specific data the relational function is usually independent of time. The mortality improvements for the specific population are consequently expressed as a static function of the mortality improvements of the general population, assuming a similar evolution of the mortality improvements between both populations. Some limitations can also be pointed out as the fact that, depending on the quality of the specific data, the model could better fit on some ages and years rather than others, meaning that some deviations will still be observed.

For more information about adverse selection, please feel free to read the White Paper

**“A Reactfin White Paper in Life, Health and Pension:
Adverse selection in lifetables models, Methods & applications”**

by Julien Antunes Mendes and Aurélie Miller



Case study
Annuities for Belgians aged 60 or more

In the case of the more than 60-years old Belgian males insured population, one can determine for example the insured population mortality surface thanks to a linear regression on the logarithms of the forces of mortality between general and insured populations. This linear relationship is used to translate the mortality surface of the general population (see Figure 2) in order to define the mortality surface of the insured population (see Figure 3).

One must take into account that this model is based on strong assumptions. Due to paucity of data, the linear relationship does not depend on time and age, meaning that only two parameters have been calibrated in order to adjust the prospective table. Some ages and times windows will therefore be better adjusted than other ones. Moreover, this linear relationship has been defined for the current year only and is applied to the whole prospective table. A stable relationship is thus assumed between the mortality improvements of the general and insured population.

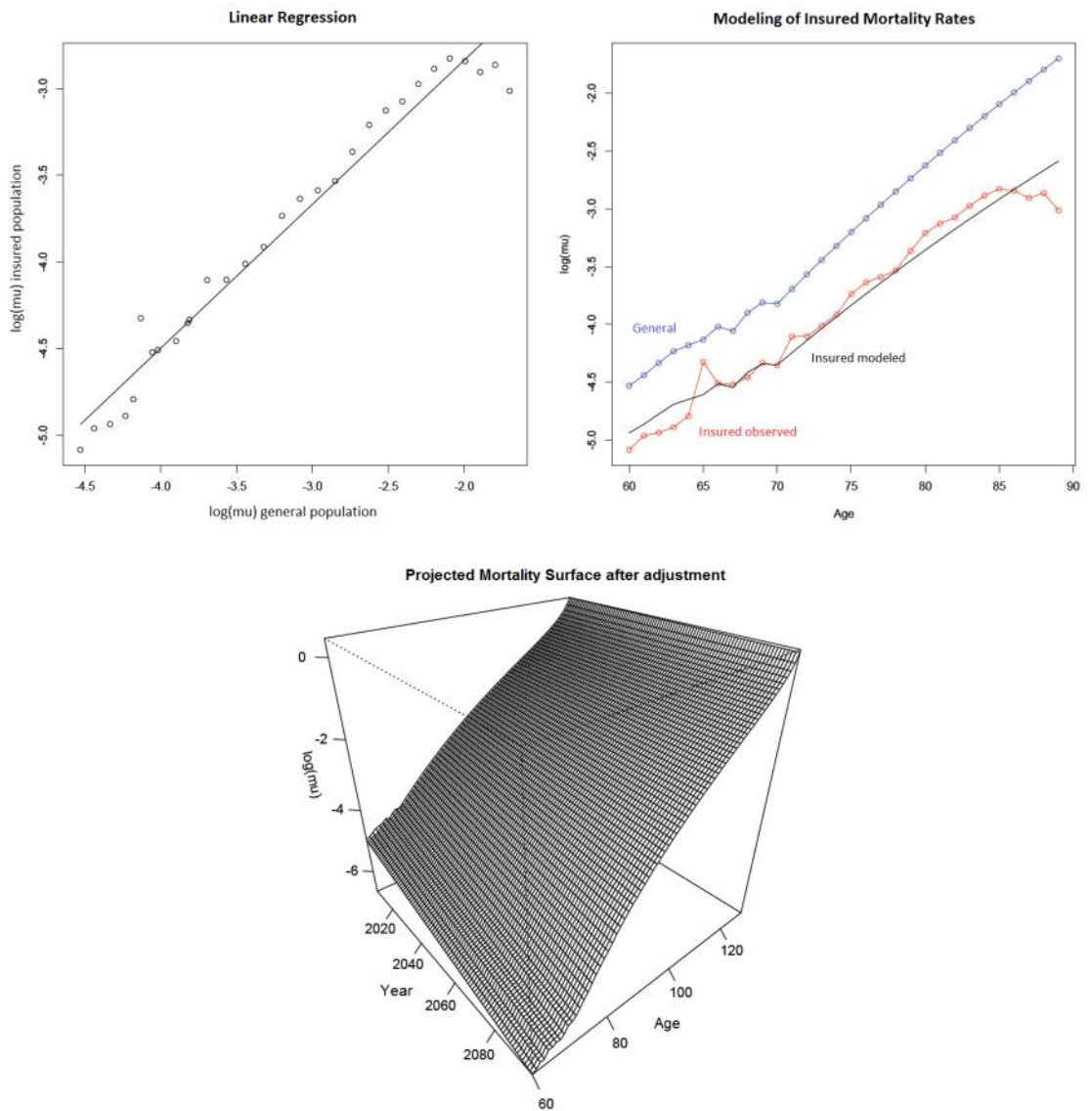


Figure 3: Adverse selection model

MORTALITY AND LONGEVITY RISK ASSESSMENT

All these presented prospective life tables' models are stochastic models. In a first step, that kind of model mainly focuses on trend risk that often appears as the most material. However, volatility around the estimation and projection of the parameters can be measured in order to draw a whole distribution of the mortality process.

One of the next interesting uses of these kinds of models could be to build an internal mortality model around these tables in order to better assess the risks and the shocks defined in the Technical Specification of the Solvency II standard model. Reacfin is currently working on this topic and will present some results in the near future.



CONCLUSION

Based on the observed mortality, prospective life tables' models are developed to estimate the evolution of the future mortality.

The construction of these kinds of models is mainly based on the general population data. However, in order to get an estimation of the mortality rates of the undertaking's portfolios, insured population data could be used, under several assumptions and limitations, to adjust the model developed at general population level. The main assumption of the presented adverse selection model is that the evolution (with time) of the mortality of the specific population is similar to the evolution of the mortality of the general population, even if the observed rates are not equal.

Developed models are based on assumptions and it is of great importance to check all of them to obtain reliable results. As far as adverse selection is considered, mortality features are different from one insurance undertaking to another – and from one portfolio to another – so that expert judgment is needed and models may be adapted in order to better reflect these specificities.

Of course, prospective life tables give an estimation of the future mortality based on assumptions and historical data but these results have to be considered carefully as deviations with reality will obviously appear. Frequent life tables updates are thus advised.

These models are by definition also stochastic models laying consequently a first stone for the assessment of longevity and mortality risks.

APPLICATION

Reacfin has developed an automated Lee-Carter tool with a dynamic and easy-to-use interface.

Based on the general population database given as input, the tool will build a complete prospective life table with many different treatments. The tool is very flexible as the user can choose many parameters and make different choices. The different steps performed are:

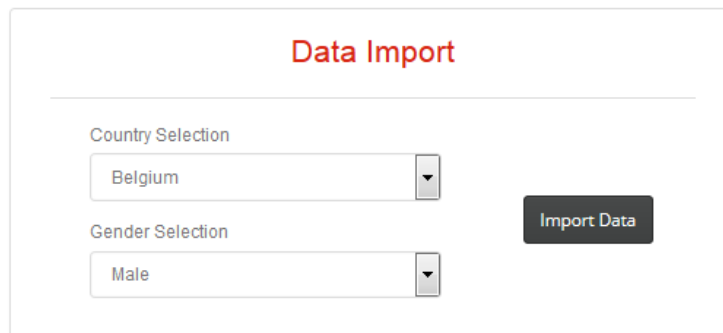
- Optimal data smoothing: Possibility to smooth the crude data in order to remove volatility
- Lee-Carter calibration: Determination of the Lee-Carter coefficients thanks to the Goodman algorithm
- Coefficients smoothing: Possibility to smooth the Lee-Carter coefficients
- Projection in the future: Extrapolation of the Lee-Carter time coefficient
- Closure at high ages: Determination of the mortality rates at high ages for which data are not very volatile
- Robustness analysis: Comparison of the Lee-Carter coefficients with two different time windows
- Backtesting analysis: Comparison of the life expectancies computed on reduced time windows

These features are illustrated here after on a simple example.

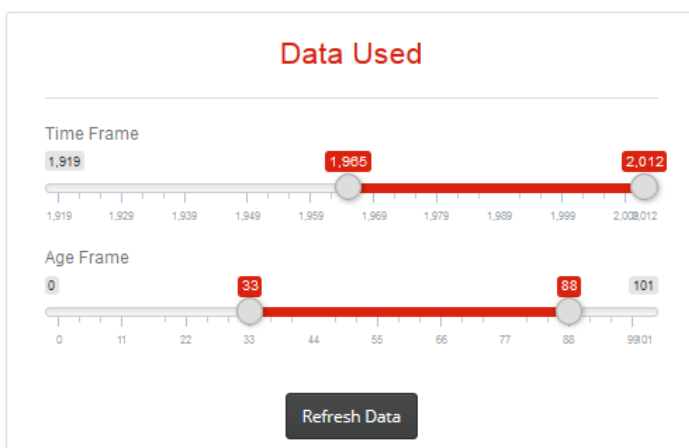
Data import

First of all, input data have to be downloaded. These are typically the historical number of deaths and exposure to risk for a given country.

In this example, the Belgian historical data from 1919 until year 2012 for males are considered.



Data selection

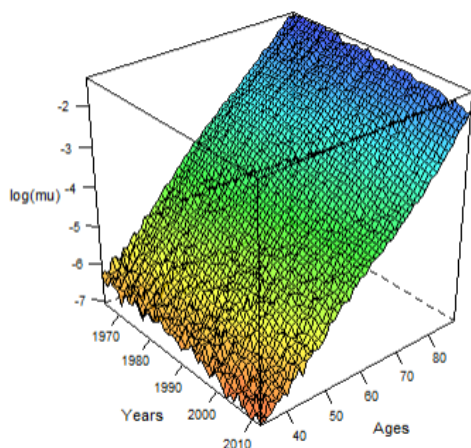


The user can restrict the age frame as well as the time frame. This is useful if the user does not need any information about certain ages (e.g. for ages higher than 65 when modelling pension products) or if too old data are not required.

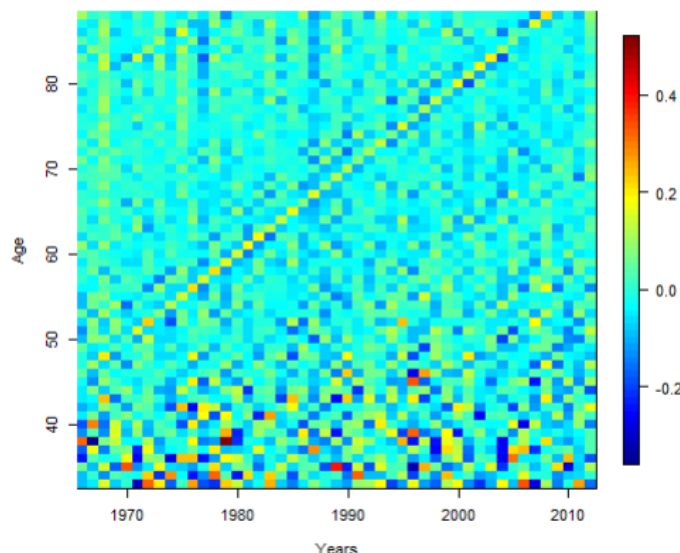
In this case, one takes into account years from 1965 to 2012 and ages from 33 to 88.

Forces of mortality as well as annual returns by age are then given.

Forces of Mortality Surface



Annual Returns by Age



Data smoothing

The user chooses if data smoothing is necessary or not. The smoothing method consists in performing local linear regressions depending on a certain neighbourhood factor. The optimal factor regarding General Cross Validation (GCV) is automatically computed by the tool. Forces of mortality before and after smoothing are depicted.

Smoothing Parameters

Perform data smoothing ?

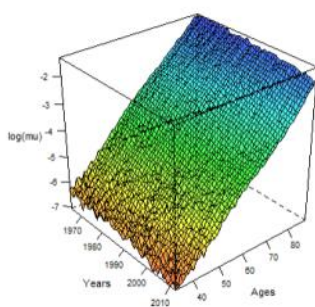
Smoothing Methodology

Smoothing Parameter

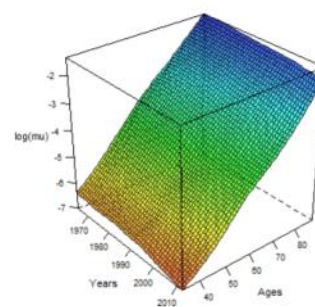
The optimal smoothing parameter is 6 %

Smoothing

Forces of Mortality Surface



Smoothed Forces of Mortality Surface



Calibration

The basis Lee-Carter is calibrated and the different coefficients $\beta_x^{(1)}$, $\beta_x^{(2)}$ and $\kappa_t^{(2)}$ are thus estimated thanks to the Goodman algorithm. To recall, the basis Lee-Carter is a model of the following form:

$$\ln m(t, x) = \beta_x^{(1)} + \beta_x^{(2)} \kappa_t^{(2)}$$

Prospective Life Tables

An improved mortality model

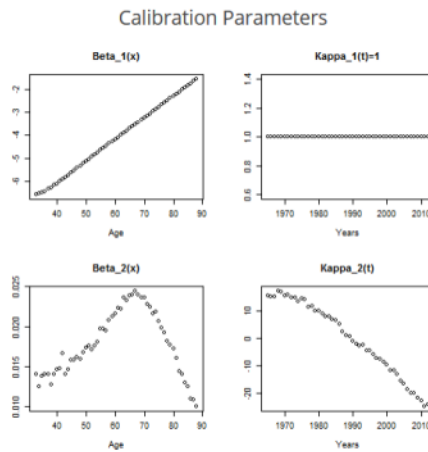
The calibrated coefficients, the resulting forces of mortality surface as well as some calibration values as the log-likelihood are given.

Calibration Parameters

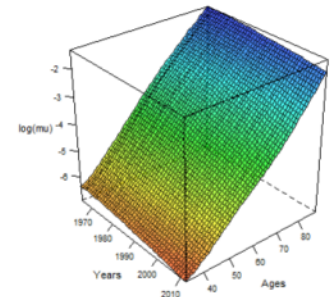
Calibration Methodology

Calibration

	Values
Log-likelihood	-9994597.43
BIC	-9995229.15
Adjusted variance of residuals	2.55



Calibrated Forces of Mortality Surface



Moreover, robustness analysis can be performed, meaning that the calibrated coefficients based on two different time frames (and with a specific age frame that can be different from the one chosen for calibration) are compared in order to determine if the shape is the same or not. Note that the trend has to be compared and not the values themselves. For example, for an age frame from 20 to 65 and two different time frames from 1965 to 2012 (black dots in the following figure) and from 1980 to 2012 (red line in the following graph), the comparison is given as follows:

Reacfin

Life Tables Toolkit

Import Data Data Selection Data Smoothing **Calibration** Coefficients Smoothing Projection Closure Backtesting Exports

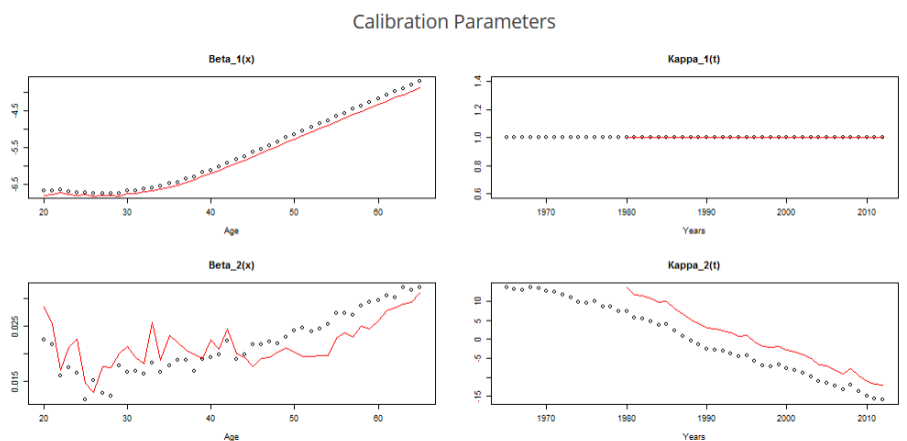
Robustness Data

Age Frame

Time Frame 1

Time Frame 2

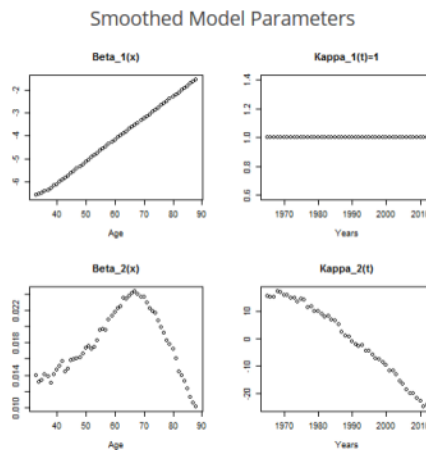
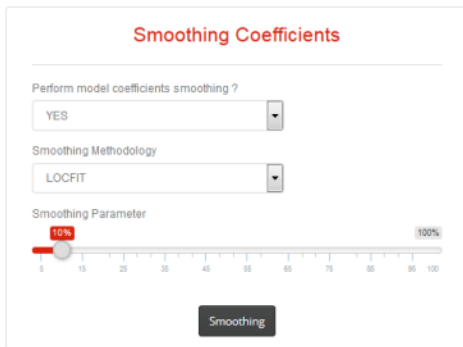
Robustness



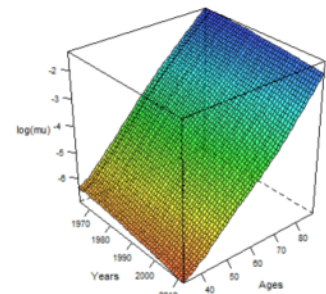
Coefficients smoothing

Once Lee-Carter coefficients are calibrated, these can also be smoothed. The smoothing method is the same as for the data smoothing and consists in performing local linear regressions depending on a certain neighbourhood factor.

Smoothed coefficients and smoothed forces of mortality are depicted.



Smoothed Forces of Mortality Surface

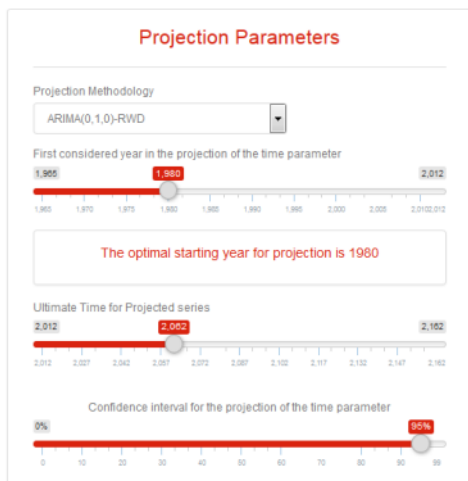


Projection

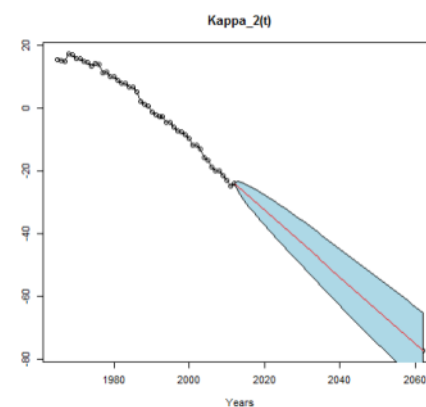
Models are calibrated in order to be able to project mortality rates in the future and thus to have an expectation of the future rates that will be used in the different analyses (reportings, profitability analysis...). The time coefficient $\kappa_t^{(2)}$ is projected thanks to an ARIMA(0,1,0) with drift. The user has to choose:

- The first considered year in the calibration of the projection: based on a determination coefficient R^2 of a linear regression, an optimal year is automatically computed by the tool. From this year until the last selected year (2012 in this example), the calibration of the ARIMA(0,1,0) with drift is performed.
- The ultimate projection year
- The confidence interval for the projection (the confidence interval is computed in a theoretical way given the Normal distribution of the projection error term)

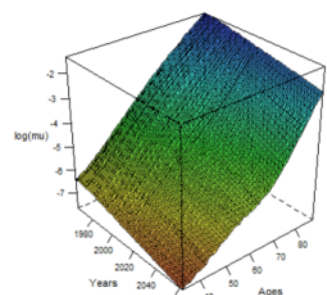
With calibration from 1980, an ultimate projection year of 2062 and a confidence interval of 95%, the projected time component and the resulting forces of mortality surface are as follows:



Projected Time Parameters



Projected Forces of Mortality Surface



Moreover, based on a certain number of simulations, the user can generate different scenarios and download the lower and upper bounds of a certain determined confidence interval.

With calibration from 1980, an ultimate projection year of 2062 and a confidence interval of 95% and 1000 simulations, the projected time component and the observed confidence interval is given here after.

Projection Parameters

Projection Methodology: ARIMA(0,1,0)-RWD

Number of simulations: 1,000

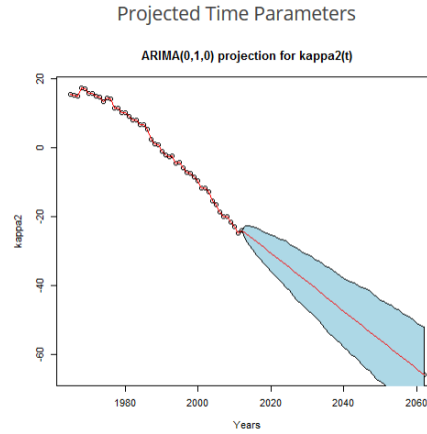
First considered year in the projection of the time parameter: 1,980

The optimal starting year for projection is 1980

Ultimate Time for Projected series: 2,062

Confidence interval for the projection of the time parameter: 95%

Projection



Closure

At high ages, observed mortality rates are often not reliable due to the lack of data implying an important volatility. To tackle this issue, closure models are implemented. The user can choose between:

- Denuit-Goderniaux model
 - Fixed initial age for closure
 - Adapted initial age for closure
- Kannisto model
- Free ultimate age model

When opting for Denuit-Goderniaux model with fixed initial age for closure at 88 and an ultimate age of 130, the forces of mortality surface before and after closure are depicted below.

Closure Parameters

Perform table closure?: YES

Closure Methodology: Denuit-Goderniaux

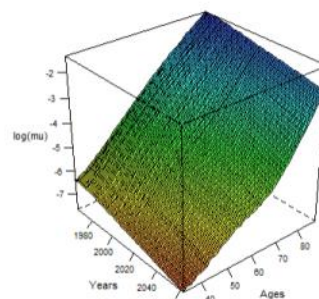
Determination of the initial age for closure: Fixed

Initial Age for Closure model: 88

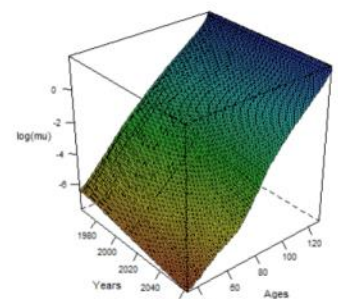
Ultimate Age: 130

Closure

Forces of Mortality Surface



Closed Forces of Mortality Surface



Backtesting

In order to assess the predictive power of the model, one can perform backtesting: calibrate the Lee-Carter model on a reduced time frame (the last selected year have to be different from the last year for which data are available), project the mortality rates and compare them with the observed ones. In this example, here are the chosen parameters:

- The period life expectancy will be backtested (and not the prospective life expectancy which can also be chosen in the tool).
- The remaining life expectancy at age 40 is considered.
- The three time frames considered begin in 1965 and end in 1977, 1988 and 2000.
- 95% confidence intervals are depicted.

The results are given here after.

Backtesting Parameters

What kind of life expectancy?

Period ▼

Life expectancy determined at that age

33 40 88

1965 1970 1975 1980 1985 1990 1995 2000 2005 2010/2012

Last calibration year 1

1965 1977 2012

Last calibration year 2

1965 1988 2012

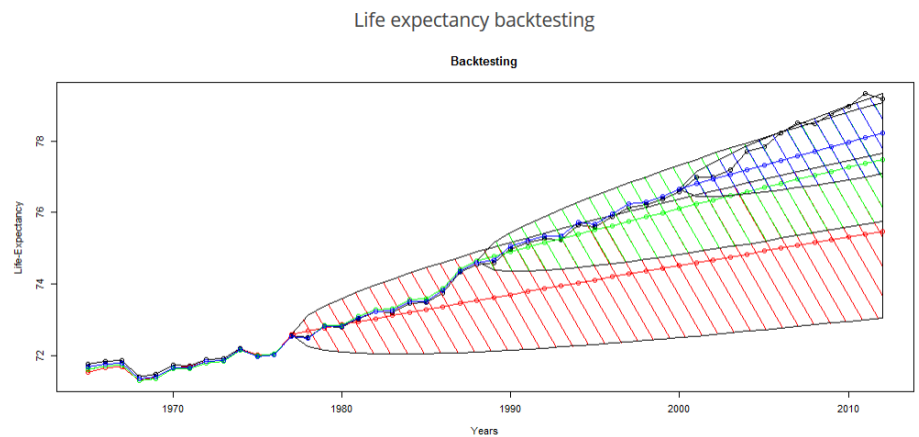
Last calibration year 3

1965 2000 2012

Confidence interval for the backtesting

0% 95%

Backtesting



Exports

Once the different steps have been performed, the user can easily download different outputs

- The resulting life table obtained after having performed each step (except the direct output steps).
- The list of all reduction rates. This permits to have a table with coefficients used to directly transform the last observed/modelled mortality rates into future mortality rates.
- A pdf report containing a summary of the Lee-Carter analysis.
- A table containing the different choices made by the user.
- A table containing the different parameters chosen by the user.

Life Table Download

Reduction Rates Download

PDF Report

Please insert comments to add to the report

Type here

Insert Comments

Download

Models choices

User Choices	
Country	Belgium
Gender	Male
Perform Data Smoothing?	NO
Data Smoothing model	2DADICFT
Smoothing coefficients Smoothing?	YES
Confidence Smoothing model	LOCKRT
Calibration model	Lee-Carter
Projection model	APMAN(1,0,RRW)
Perform closure?	YES
Closure model	Demai-Godemaou

Download

Parameters choices

User Choices	
Min age for calibration	33
Max age for calibration	88
Min year for calibration	1965
Max year for calibration	2012
Smoothing data parameter (k)	4
Smoothing coefficients parameter (k)	10
Initial year for projection	1965
Maximal year for projection	2012
Wilksens Year for projection	2042
Starting age for closure	75
Closure age for closure	130

Download

HOW REACFIN CAN HELP

Reacfin has already built many prospective life tables for most of its clients and several of its consultants have gained substantial experience for this topic.

As described in the previous section, Reacfin has recently spent substantial efforts in developing a tool aimed at supporting our clients projecting mortality in the future in an easy and understandable way.

A few examples of practical interventions include:

- Deployment of the tool and possible tailoring
- Construction of an entire prospective life table using state-of-the-art methods
- Supporting our clients constructing such tables
- Validation of a life table, including benchmarking with other possible methods and backtesting
- Review of the construction of a life table
- Development of an adverse selection model to capture specific mortality trends (e.g. insurance portfolio mortality)

We are convinced that this tool perfectly matches the insurance companies' needs and this could be a powerful, easy and quick way to build life tables.

If you want to investigate this topic further, please visit our [website \(www.reacfin.com\)](http://www.reacfin.com), tab **OnlineApp**. On the page dedicated to the Life Prospective Mortality Tool, you will find a light demo version of the tool.

In this demo version, limited historical data until 1995 for six different European countries (Belgium, France, Germany, Luxembourg, Slovenia and The Netherlands) can be chosen. A more complete version is available on demand.

Note that the output life table of this tool can be used as inputs of the "Adverse Selection tool" developed by Reacfin to build life tables adapted to a specific population (insurance portfolio, product portfolio...). For more information about this tool, please feel free to read the White Paper "A Reacfin White Paper in Life, Health and Pension: Adverse selection in lifetables models, Methods & applications" by Julien Antunes Mendes and Aurélie Miller.

If you have questions about this tool or if you are interested by the full version, please feel free to contact us: OnlineApp@reacfin.com.

About Reactfin

Reactfin is a consulting firm focused on setting up best quality tailor-made Risk Management Frameworks, and offering state-of-the-art actuarial and financial techniques, methodologies & risk strategies.

While we initially dedicated ourselves to the financial services industry, we now also serve corporate or public-finance clients.

Advancements in finance and actuarial techniques are developing at a fast pace nowadays. Reactfin proposes highly-skilled and experienced practitioners, employing innovative techniques and offering expertise in compliance and risk strategies & governance. Our support will allow your firm to reach top performances and gain new competitive advantages.

As a spin-off from the UCL (University of Louvain which ranks eighth in the world formaster's degrees in insurance), we maintain a strong link with this institution which enables us to give independent, tailored and robust advice on risk management, actuarial practices and financial models.

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